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Compressive complex wave retrieval from a single off-axis digital Fresnel hologram for quantitative phase imaging and microlens characterization

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ABSTRACT

A compressive sensing (CS) approach is proposed for the complex wave retrieval algorithm from single off-axis digital Fresnel holography to improve the accuracy of quantitative phase measurement. The linear model of non-linear holographic process used in the complex wave retrieval algorithm is utilized to meet the linearity requirement of the proposed CS implementation presented in this paper. The error in object wave reconstruction due to the approximations used in complex wave retrieval algorithm is compensated by combining it with CS approach. A Mach–Zehnder interferometric geometry in transmission and off-axis mode in the Fresnel domain is used to record the digital hologram. The numerical simulations and experimental results are presented to demonstrate and validate the proof of the concept and to compare CS frame work with conventional complex wave retrieval algorithm. The proposed algorithm is tested for the microlens sample characterization by considering the parameters such as height, diameter and radius of curvature. A Haar wavelet based object wave sparsification is used in the experiment for the prudent phase reconstruction in the CS approach. The proposed method enables an improved and accurate object phase reconstruction, with minimal noise such as distortions or speckles with respect to the conventional complex wave retrieval method.

1. Introduction

Compressive sensing (CS) [1,2] is an innovative mathematical framework and a paradigm shift in signal processing algorithm development. This method enables the accurate reconstruction of the original signal from a fewer samples obtained by a linear observation of the signal. The CS frame work suggests that the number of observed samples required for the faithful reconstruction of the original signal is lesser than the Nyquist rate. The framework also compensates for the noise in the observation. This is achieved by exploiting the sparsity of the original signal and by solving the undetermined linear equations by considering the linearity of the measurement model of the samples. The linearity of the measurement model is the key requirement for the application of CS frame work. Any sensing process, based on the linear observation, that involves noise and incompleteness shall be compensated by CS to accurately reconstruct the sparse original signal. If the original signal is not sparse in time or spatial domain, then the suitable sparsifying operators like wavelets, Fourier transforms and other frequency transforms are applied to represent the signal sparsely. These features make CS a good choice for data compression in many signal and image processing applications [3–6]. The capability of CS to reconstruct the original signal from an incomplete noisy measurement makes it a suitable choice for digital holography in the mathematical models, which involve many approximations and are thus incomplete in nature. Compressive digital holography has attracted many researchers and several works have been reported in this emerging topic [7–23]. The compressive digital holography is applicable only if the object wave is either sparse in the spatial domain or is compressible in some transform domains like FFT, DWT, etc. It also helps in the estimation of the original object wave from an incomplete and noisy linear measurement by an iterative procedure [2,3].

Digital holography is an imaging tool [24–32] for the quantitative measurement of amplitude and phase of the object wave. The recorded digital hologram is a two-dimensional real valued image that contains the three-dimensional information of the object. The digital hologram is numerically reconstructed to obtain the 2D complex image [33–37] which contains the three-dimensional object information. The amplitude of the complex information gives the intensity information and the phase part gives the depth or three-dimensional information of the object. This information has significant implications in phase contrast imaging and three-dimensional information processing. All holographic model equations comprise of approximations and the object field determined from these holograms is considered as noisy measurement of the original object wave. When CS is applied in the reconstruction process with an appropriate sparsity criterion, an accurate complex object wave

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reconstruction is possible from the noisy measurements. It is found in the literature that the CS has been applied to the reconstruction of object wave using single exposure in-line holography by modelling the non-linear in-line holography formulation as a linear process and there are several works reported in this direction [7–9]. In this approach, the linearity is achieved by assuming the non-linear term as a model error. This is a valid approximation to achieve the linearity requirement for the CS implementation, which at the same time could affect the accuracy of whole information reconstruction, especially the phase of the object wave. Moreover, the aspect of phase reconstruction ability of digital holography is not found discussed much in many of the CS based approaches in digital holography. Hence an attempt has been made in this paper to address the phase reconstruction through compressive digital holography. The classical phase-shifting digital holography (PSDH) [38,39] is a linear process and suitable for the application of CS algorithm. The CS application in PSDH is demonstrated to reconstruct the good quality images from a lesser number of detection points in the Fresnel field [14]. Though the reconstruction accuracy is better in PSDH, it requires recording of multiple holograms using different reference wave phase shifts, and therefore, it cannot be used for phase imaging of the dynamic objects.

The complex wave retrieval algorithm from a single off-axis digital Fresnel hologram proposed by *Liebling* et al. [40] is a non-linear approach and has been successfully used for object wave reconstruction in various applications of digital holography [28,41–44]. By a practical assumption that the amplitude of the reference wave is greater than that of the object wave and by adopting a non-linear change of variables, the non-linear holography is approximated to a linear process that can be solved by linear algorithm to determine the Fresnel field. In the crux of this algorithm, the non-linearity boils down to the linearity. The method is demonstrated to reconstruct both the intensity and the phase of the object wave but it is shown that there is a deviation in phase reconstruction accuracy due to the approximations. Hence we explore the possibility of using this linear model as a sensing matrix in the proposed compressive off-axis digital Fresnel holography to improve the accuracy of whole information reconstruction. Thus the linearity of the complex wave retrieval algorithm from a single shot off-axis digital Fresnel hologram is very apt for CS framework. The main idea of this paper is to develop a compressive single shot off-axis digital holographic scheme for accurate complex object wave reconstruction without discarding the non-linearity of the holographic process with practical assumptions. Moreover, the use of single shot scheme is important for observing the dynamic events using digital holography.

The proposed CS approach is implemented in this paper using the above mentioned complex wave retrieval as sensing matrix with Haar wavelet sparsification. The numerical simulations have been carried out to show its merits over the conventional complex wave retrieval method in terms of reconstruction accuracy of both intensity and phase, followed by the experimental validation of the results. The experimental validation of results was carried out on a microlens object using the Mach–Zehnder interferometric recording geometry for single off-axis digital holography. The microlens characterizations like height (h), diameter (D) and radius of curvature (ROC) were performed using conventional and CS based method and compared. The analysis of the results obtained has revealed that the characterization of CS based method is far superior to that of conventional method. The CS algorithm of the proposed method was implemented by using the Gradient Projection for Sparse Reconstruction (GPSR) algorithm [3].

2. Proposed compressive sensing model for single off-axis digital holography

Let $O_o\left(x_o, y_o\right)$ be the complex object wave and $U\left(x, y\right)$ be its Fresnel field (Fresnel plane) at a distance 'd' from the object plane. They are related by the following equation:

$$U\left(x, y, d\right) = \frac{e^{ikd}}{i\lambda d}\iint O_o\left(x_o, y_o\right)\exp\frac{i\pi}{\lambda d}\left[\left(x-x_o\right)^2+\left(y-y_o\right)^2\right]dx\,dy \quad (1)$$

For simplicity, Eq. (1) can be written as in Eq. (2).

$$U\left(x, y\right) = \mathcal{F}_d\left[O_o\left(x_o, y_o\right)\right] \quad (2)$$

where $\left(x_o, y_o\right)$ and $\left(x, y\right)$ are the co-ordinates of the object plane and Fresnel plane respectively. The term, $\mathcal{F}_d$ is a linear operator that performs Fresnel transform of the object field for a propagation distance d. Here, both $U\left(x, y\right)$ and $O_o\left(x_o, y_o\right)$ are the complex numbers.

A digital Fresnel hologram is formed by the superposition between the Fresnel field $U\left(x, y\right)$ and off-axis plane reference wave $R\left(x, y\right)$. Let $U\left(x, y, d\right) = O_u exp(i\vec{k}_1.\vec{r})$ and $R\left(x, y\right) = exp(i\vec{k}_2.\vec{r})$. The single shot off-axis digital Fresnel hologram $H(x, y)$ is described by Eq. (3).

$$\begin{aligned} H\left(x, y\right) &= \left|U\left(x, y\right) + R\left(x, y\right)\right|^2 \\ &= \left|O_u exp(i\vec{k}_1.\vec{r}) + exp(i\vec{k}_2.\vec{r})\right|^2 \end{aligned} \quad (3)$$

here $H\left(x, y\right)$ is the measured intensity of the digital Fresnel hologram with sampling period ΔM, and $\vec{k}$ is the propagation vector. Now the resultant wave vector $\vec{K}$ of the fringe pattern is given by

$$\left|\vec{K}\right| = \left|\vec{k}_1 - \vec{k}_2\right| = \frac{4\pi}{\lambda}\sin\left(\frac{\theta}{2}\right) \quad (4)$$

where θ is the angle between the interfering beams.

Using computational techniques, U the Fresnel field $U\left(x, y\right)$ at the detection plane can be retrieved from the recorded single off-axis digital hologram by a non-linear complex wave retrieval algorithm proposed by *Liebling* et al. [40]. The non-linear change of variables adopted in this method could transform the non-linear holography problem that can be solved by a linear algorithm. This transformation is achieved by a practical assumption that the amplitude of the reference wave is greater than that of the object wave. Also the local least square estimation of amplitude and phase of reference wave is carried out by assuming an *a priory* model of the reference wave phase. In this algorithm, the Fresnel field $U\left(x, y\right)$ at the acquisition plane is obtained by solving a non-linear set of equations in least square sense. The assumptions and other lower order approximations used in this method can lead to a deviation in the obtained phase with respect to the original phase and may be termed as incomplete Fresnel field. The use of this incomplete Fresnel field information will lead to noise in the reconstructed complex object wave field. For quantitative phase contrast imaging and object characterization, a good quality phase map of the reconstructed object field is highly expected. To recover the accurate complex object field from an originally retrieved incomplete Fresnel field, CS is applied to the complex wave retrieval and object wave reconstruction algorithms of off-axis single shot digital Fresnel holography. For the implementation of CS algorithm, a new approximated Fresnel transform operator, $\tilde{\mathcal{F}}_d$ and noisy Fresnel field, $\tilde{U}\left(x, y\right)$ are introduced so that an ill-conditioned version of $U\left(x, y\right)$ denoted by $\tilde{U}\left(x, y\right)$ is obtained.

$$\tilde{U}\left(x, y\right) = \tilde{\mathcal{F}}_d O_o\left(x_o, y_o\right) \quad (5)$$

Thus, the entire process computing the Fresnel field from the single off-axis digital Fresnel hologram is denoted as the operator $\tilde{\mathcal{F}}_d$. Now, the reconstructed Fresnel field at the detector plane is inverse Fresnel transformed to get the approximated complex object information at the object plane as shown in Eq. (6).

$$O_o\left(x_o, y_o\right) = \mathcal{F}_d^{-1}\left[\tilde{U}\left(x, y\right)\right] \quad (6)$$

Here, Eq. (2) and Eq. (6) are linear so that CS can be applied to improve the reconstruction accuracy of the object field. The CS is a novel mathematical framework and can be applied on noisy incomplete measurements to reconstruct the sparse complex input object wave $O_o\left(x_o, y_o\right)$ by solving the unconstrained optimization problem [3] as shown in Eq. (7).

$$\underbrace{min}_{O_o\left(x_o, y_o\right)}\frac{1}{2}\left\|\tilde{U}\left(x, y\right) - \tilde{\mathcal{F}}_d O_o\left(x_o, y_o\right)\right\|_2^2 + \tau\left\|O_o\left(x_o, y_o\right)\right\|_1 \quad (7)$$

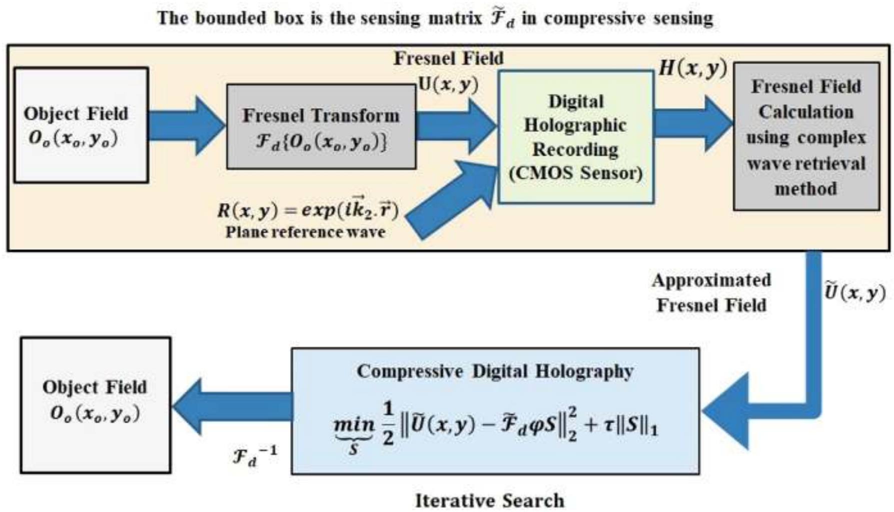

Fig. 1. Proposed compressive complex wave retrieval method for single off-axis digital Fresnel hologram.

where τ is a non-negative parameter. This equation is formulated using the quadric programming and is minimized iteratively by using Gradient projection sparse representation (GPSR) algorithm [3], which uses the forward transform $\tilde{\mathcal{F}}_d$ and its inverse $\tilde{\mathcal{F}}_d^{-1}$ as operators. In each iteration, the search path is determined by projecting the negative gradient in the feasible set. Here in Eq. (7), it is considered that the complex object wave is sparse in the spatial domain. Assuming that $O_o(x_o, y_o)$ is not sparse in the spatial domain but sparse in the transform domain such that $O_o(x_o, y_o) = \varphi S$, where S is sparse in nature and φ is a basis function of the transform. Now Eq. (7) can be rewritten as

$$\underbrace{min}_{S} \frac{1}{2}\left\|\tilde{U}(x, y) - \tilde{\mathcal{F}}_d \varphi S\right\|_2^2 + \tau \|S\|_1 \tag{8}$$

where $\tilde{\mathcal{F}}_d$ and φ indicates the sensing and sparsifying matrices respectively. These matrices should be mutually non-coherent for the faithful reconstruction of signal $O_o(x_o, y_o)$. Now the operators for GPSR are $\tilde{\mathcal{F}}_d\varphi$ and $\tilde{\mathcal{F}}_d^{-1}\varphi^{-1}$. An optimal solution for the object field which is in close agreement with the original sparse object function $O_o(x_o, y_o)$ can be searched by using an appropriate measurement model and by solving the unconstraint optimization given by Eq. (8). The minimization of Eq. (8) is achieved by performing following steps in the implementation.

(i) The noisy Fresnel field $\tilde{U}(x, y)$ obtained by conventional complex wave retrieval method is used as the noisy measurements in the GPSR implementation.
(ii) GPSR implementation [3] using MATLAB is executed by selecting arbitrary value of τ in Eq. (8).
(iii) The forward transform used in GPSR is set as $\tilde{\mathcal{F}}_d\varphi$, φ is the Haar wavelet reconstruction and $\tilde{\mathcal{F}}_d$ is mathematical process of obtaining the noisy Fresnel field using off-axis digital Fresnel holography and complex wave retrieval method.
(iv) The reverse transform used in GPSR is set as $\tilde{\mathcal{F}}_d^{-1}\varphi^{-1}$ in which $\tilde{\mathcal{F}}_d^{-1}$ is the inverse Fresnel transform and φ^{-1} is Haar wavelet decomposition.
(v) The obtained result after convergence is verified and τ is varied using trial and error method to get the best performance.

The Eq. (8) aims to search for an accurate complex input object wave and denoise the complex object wave in an efficient manner. The schematic of the proposed compressive complex wave retrieval method for single off-axis digital Fresnel hologram is shown in Fig. 1. In Section 3, we present the mathematical model for the proposed compressive complex wave retrieval from a single off-axis digital Fresnel hologram.

3. Implementation of sensing and sparsifying matrices for complex wave retrieval algorithm

The requirements of sensing operator in CS is given in Eq. (2) and Eq. (6). The sensing operator in the proposed CS scheme is a Fresnel transform and it is a linear operator. Thus, the method satisfies the requirement of linearity in the implementation of CS in complex wave retrieval from single off-axis digital Fresnel hologram. The Haar wavelet is used as a sparsifying operator in the place of φ in Eq. (8). In the complex wave retrieval method, the mathematical model for calculating the Fresnel field [40] at the detector plane from a single shot off-axis hologram is given in Eqs. (9)–(12).

$$\begin{bmatrix} 1 & a & a^* \\ a & b & 1 \\ a^* & 1 & b^* \end{bmatrix} \begin{pmatrix} K \\ Z \\ Z^* \end{pmatrix} = \begin{pmatrix} \sum_m \omega_m H_m \\ \sum_m \omega_m V_m H_m \\ \sum_m \omega_m V_m^* H_m \end{pmatrix} \tag{9}$$

$$U = \frac{Z}{A} \tag{10}$$

$$A_{\pm} = \left(\frac{K \pm \sqrt{K^2 - 4Z^2}}{2}\right)^{1/2} \tag{11}$$

The window function ω_m is given in Eq. (12)

$$\omega_m(k, l) = \beta^n(k/s)\,\beta^n(l/s) \text{ with } s = (L-1)/(n+1) \tag{12}$$

The Eq. (9) yields three linear equations. By solving the non-linear Eq. (3) in the least square estimation to change non-linear holography equation into a linear variable with some new auxiliary variables K and Z. where H_m $(m = 1, 2, \dots, M)$ are the position of M pixels within the region of interest (x, y), $V_m = R_m^*/A = exp(-i\theta_m)$ which is normalized reference wave, A is the amplitude of reference wave, $a = \sum_m \omega_m V_m$ and $b = \sum_m \omega_m V_m^2$, ω_m indicate the non-negative weight, β^n denotes cubic B-spline of degree n and L is the size of the window. The algorithm uses a bell-shaped Gaussian like function which is finite and separable. The non-negative weights are normalized for far away from the point of interest i.e. $\sum_m \omega_m = 1$.

The Eq. (9) determines the unknown variables K, Z and Z^*. On substitution of K and Z in Eq. (11) to solve Eq. (10), would give the complex object wave at the hologram detection plane i.e. the Fresnel transform of the complex object wave field. Since this retrieved information at the recording plane is a Fresnel transform of the object field, an inverse Fresnel transform results in the reconstruction of the original complex object field. This is called back-propagation of the retrieved Fresnel field to the object plane from the recording plane. Using Eq.

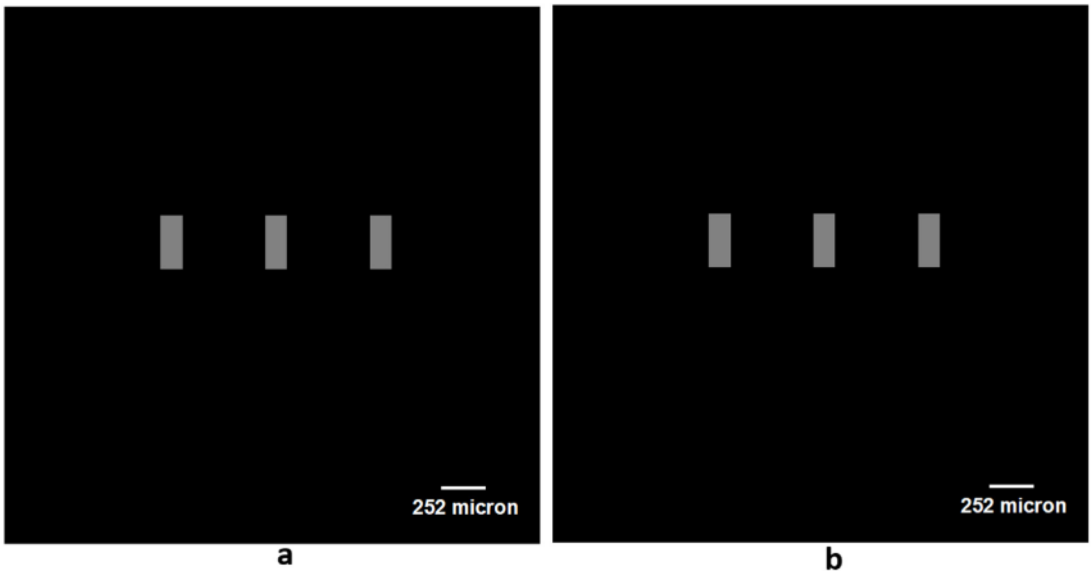

Fig. 2. Simulated input complex grating object wave (a) intensity and (b) phase.

(6), we get the approximated input object wave field $\tilde{O}_o(x_o, y_o)$ from $\tilde{U}(x, y)$. The proposed process of compressive complex wave retrieval from single shot off-axis digital Fresnel hologram can be referred from the schematic shown in Fig. 1. The entire process of applying complex wave retrieval method from single off-axis hologram to obtain the noisy approximated Fresnel field $\tilde{U}(x, y)$ at the detector plane is denoted by operator $\tilde{\mathcal{F}}_d$. The compressive complex wave retrieval method is implemented by solving Eq. (8) with Haar wavelet sparsification.

4. Characterization of the microlens using compressive digital holographic system

A microlens array is used as a specimen to test the proposed compressive complex wave retrieval method. The reconstructed phase of the sample is wrapped in the range of $[-\pi, \pi]$ due to the operation mentioned in Eq. (13). This phase is to be unwrapped using a robust 2D phase unwrapping algorithm to obtain the continuous phase map of the sample and thus to obtain the object profile. The wrapped phase, $w(x, y)$ of the microlens is calculated from the retrieved complex object field as:

$$w(x, y) = tan^{-1}\left(\frac{Im[O_o(x, y)]}{Re[O_o(x, y)]}\right) \tag{13}$$

In the present paper, we use a two-dimensional phase unwrapping method that uses fast cosine transforms and iterative methods in least square sense proposed by *Ghiglia and Romero* [45]. The surface profile of the microlens is calculated from the unwrapped phase by using the following equation [24,28].

$$p(x, y) = \frac{u(x, y)}{2\pi} \cdot \frac{\lambda}{\Delta n} \tag{14}$$

where p is the distance travelled by the laser light of wavelength λ through the microlens, $u(x, y)$ is the unwrapped phase and Δn is the refractive index change $(n - n_0)$, where n is the refractive index of the microlens material and n_0 is the refractive index of the ambient air. The refractive index of the material of the microlens, $n = 1.457$ is used in the computations. The parameters such as height (h) and diameter (D) of the microlens are computed from the surface phase profile of the microlens. Further, the radius of curvature (ROC) of the microlens is computed using Eq. (15).

$$ROC = \frac{h}{2} + \frac{D^2}{8h} \tag{15}$$

The parameters h, D and ROC can be calculated for both conventional complex wave retrieval and compressive complex wave retrieval methods to compare the accuracy of the techniques.

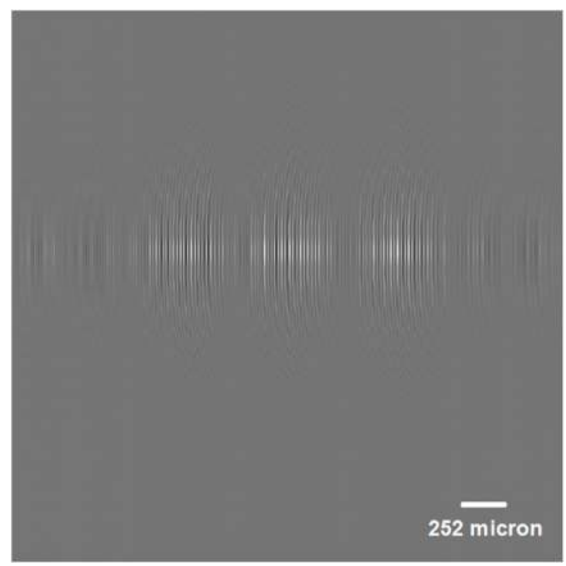

Fig. 3. Simulated single off-axis digital Fresnel hologram.

5. Numerical simulation results and discussion

The numerical simulations have been carried out to show the performance of the proposed CS method. To simulate single shot off-axis Fresnel hologram, a complex grating $O_o(x_o, y_o) = 0.5e^{j0.5}$ of size 512×512 pixels was considered as an input object wave. Each rectangular slit has a uniform size of 21×51 pixels that having a resolution of 8 lines per mm. The intensity and phase of the simulated complex grating are shown in Fig. 2(a) and 2(b) respectively.

This object field is sufficiently sparse and suitable for the CS method. The parameters used for the numerical simulation were, $\lambda =$ 632.8nm, the square pixel pitch format of the sensor $\Delta M = 6$μm and recording distance $d = 60$mm. In order to meet the requirement of complex wave retrieval method, the intensity of the object wave was chosen as 0.5 and that of reference wave was 1.0. The complex object wave $O_o(x_o, y_o)$ was propagated by a distance $d = 60$mm to the recording plane using a Fresnel transform to get the propagation field matrix $U(x, y)$. The digital hologram was obtained by the interference

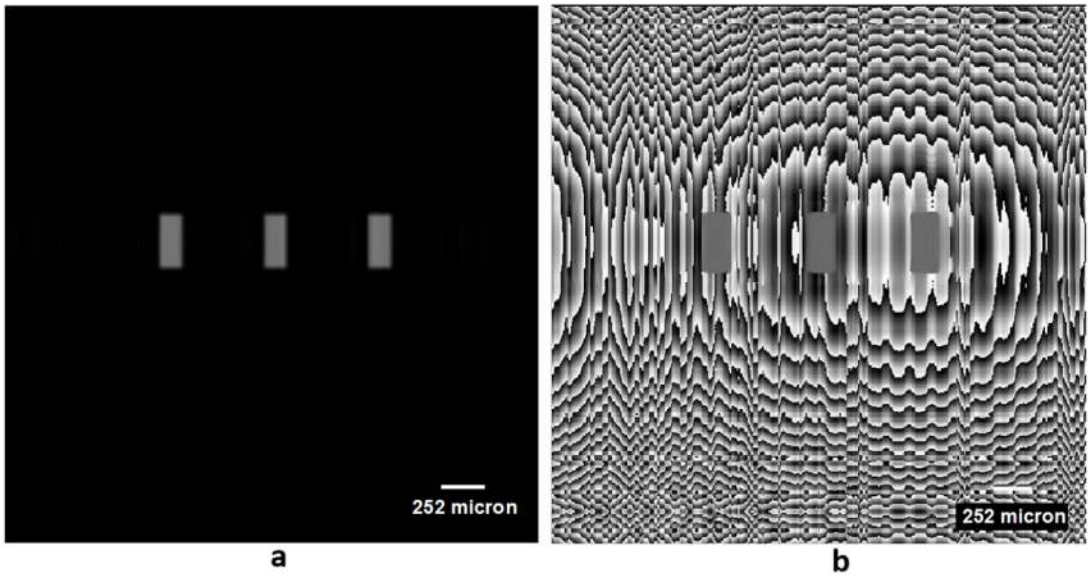

Fig. 4. Complex object wave reconstructed using the conventional complex wave retrieval method (a) intensity (b) phase.

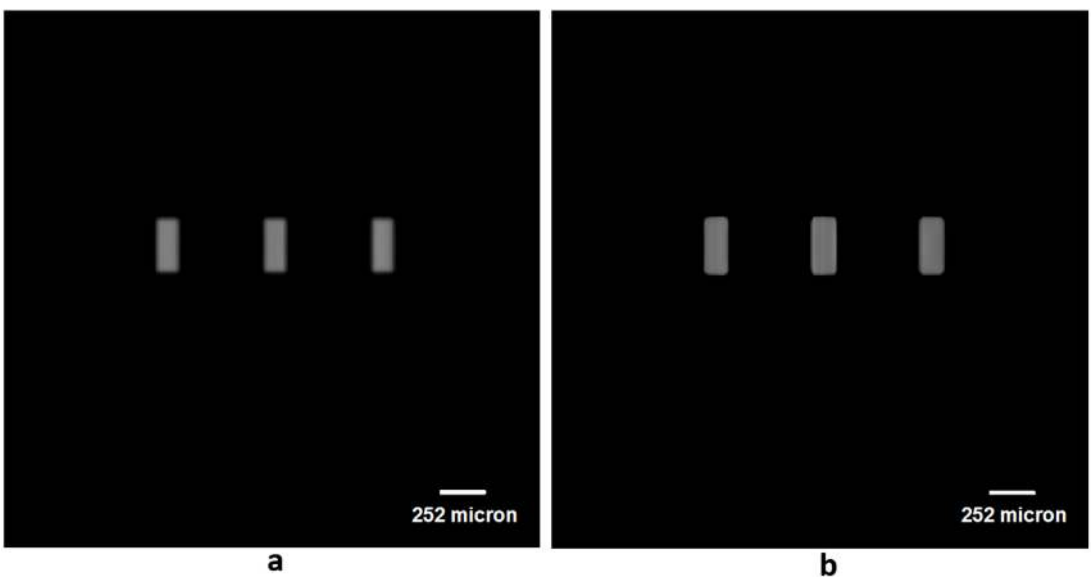

Fig. 5. Complex object wave reconstructed using the proposed compressive complex wave retrieval method with Haar wavelet sparsification (a) intensity (b) phase.

between the fields $U(x, y)$ and $R(x, y)$ with an angle $\theta = 1.22°$ which is govern by Eq. (3). The simulated single off-axis digital Fresnel hologram is shown in Fig. 3.

In order to reconstruct the complex grating object wave from the simulated digital hologram, a window size of 13×13 pixels were used at every point with a cubic B-spline given by Eq. (12). The reconstructed intensity and phase using conventional complex wave retrieval method are shown in Figs. 4(a) and 4(b) respectively. It can be observed that the reconstructed phase of the object shown in Fig. 4(b) is degraded due to the approximation used in the conventional complex wave retrieval algorithm whereas the intensity reconstruction shown in Fig. 4(a) is not much affected. This implies that the object wave $\tilde{O}_o\left(x_o, y_o\right)$ reconstructed from the Fresnel field $\tilde{U}(x, y)$ using the conventional complex wave retrieval method is an approximated version of the original object wave. Therefore, the Fresnel filed $\tilde{U}(x, y)$ is considered as noisy and incomplete measurement of the original Fresnel field $U(x, y)$ to implement the proposed CS method. An optimal solution for the object field which is in close agreement with the original sparse object function $O_o\left(x_o, y_o\right)$ can be searched by using an appropriate measurement model and by solving the unconstraint optimization given by Eq. (8). Fig. 5(a) and Fig. 5(b) show the reconstructed intensity and phase of the object wave respectively using CS method. It can be inferred from Fig. 5(b) that the quality of phase reconstruction is significantly improved by compressive complex wave retrieval algorithm with Haar wavelet sparsification of the object function. A plot comparing the reconstructed phase distribution using the above mentioned two cases with respect to the original object phase is shown in Fig. 6. The deviation from the original phase is found to be very minimal in the CS based method when compared to that of the conventional method. Thus, the compressive complex wave retrieval gives a very close approximation of the original object phase and therefore it is proven to be the best suitable method for quantitative phase imaging and phase characterization.

The complex slit function (Fig. 2) used in the previous case has uniform phase values in all the pixels in the grating region. To show the capability of the proposed CS algorithm to reconstruct the varying phase distribution, another simulation is considered in which the original input complex object wave is generated using the USAF chart of dimension 512 × 512 as intensity and a matrix of same dimension

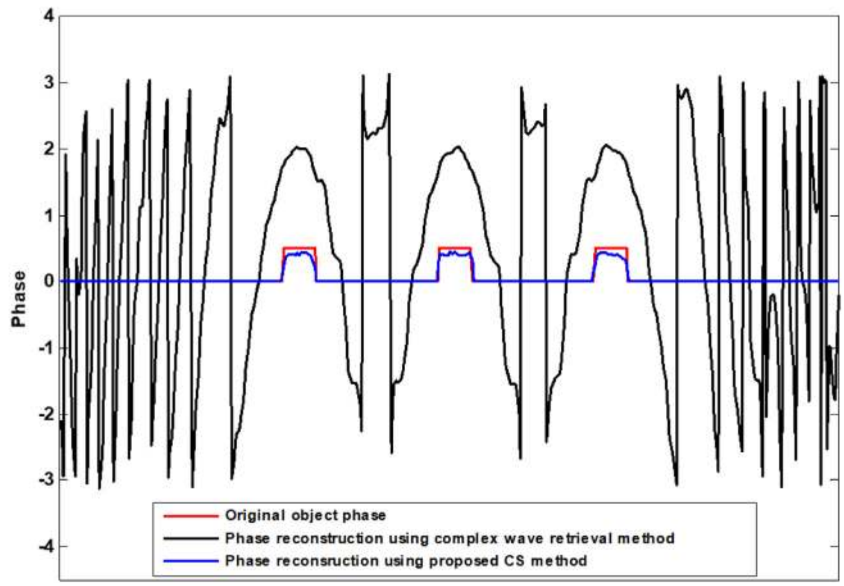

Fig. 6. Comparison of original object phase with respect to the reconstructed phase of conventional complex wave retrieval method and the proposed compressive complex wave retrieval method.

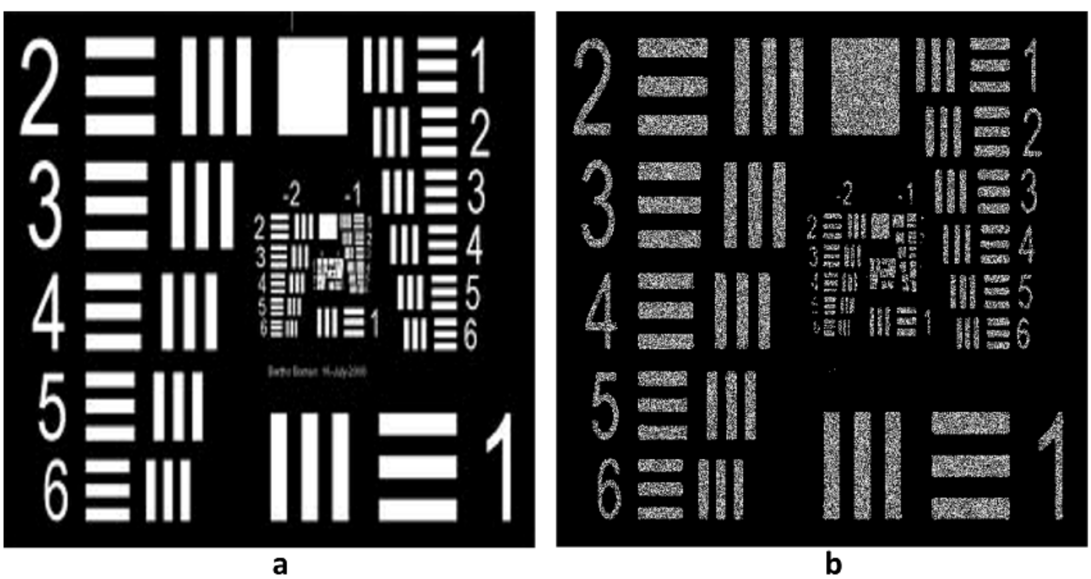

Fig. 7. Simulated input complex USAF chart as object wave (a) intensity and (b) phase.

with random values uniformly distributed in the range (0–1) as phase. The intensity and phase of the USAF chart are shown in Fig. 7(a) and Fig. 7(b) respectively. The intensity and the phase reconstructed using the conventional complex wave retrieval method are shown in Fig. 8(a) and Fig. 8(b) respectively. The intensity and phase of the complex object wave retrieved using the proposed CS method with Haar wavelet sparsification are shown in Fig. 9(a) and 9(b) respectively. To compare the reconstruction accuracy of complex wave retrieval method and the proposed CS method, the norm of the difference between the original and the reconstructed complex images were computed in each case and presented in Table 1. The results show that, norm error of the proposed CS method is very less in all the cases studied.

6. Experiment

Fig. 10 shows the schematic representation of the experimental set-up for recording an off-axis digital Fresnel hologram. It consisted of a Mach–Zehnder interferometer geometry, in which an object wave and a plane reference wave were interfering in an off-axis mode. The resulting digital hologram was detected using a CMOS sensor. A He-Ne laser emitting light of wavelength $\lambda = 632.8\text{nm}$ was used as a light source. A half wave plate was used to control the intensity of the laser beam and a spatial filter assembly (SF) was used to purify the laser beam. The collimation lens (L) of focal length FL=100 mm was used to collimate the outgoing light. The collimated beam was split into two arms using polarizing beam-splitter (PBS) and was folded using mirrors M1 and M2. The object was kept in the ARM1 in the transmission mode and ARM2 formed the reference wave. Another half wave plate was used to ensure that the object and the reference beams were in the same polarization states. The beam-splitter (BS) combines the object wave and reference wave to form the interference pattern. The angle of interference was $\theta = 3.96°$. An off-axis digital Fresnel hologram was recorded using a CMOS sensor with $6\mu\text{m} \times 6\mu\text{m}$ square pixel pitch format. The microlens sample was placed at a distance of $d = 180\text{mm}$ from the recording plane. A microlens array was used as a test object that has to be characterized using the proposed compressive complex

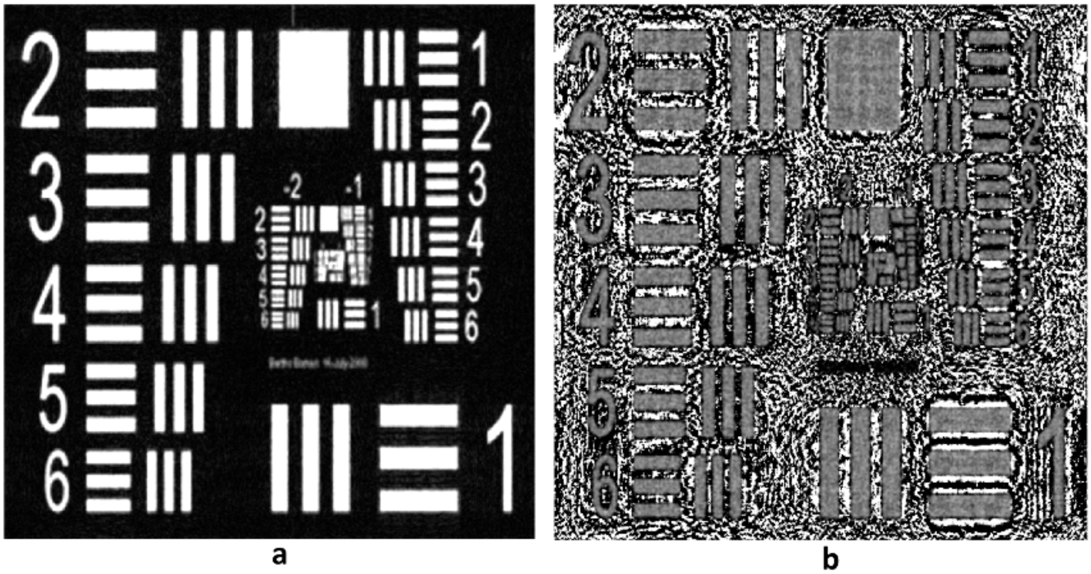

Fig. 8. Complex USAF object wave reconstructed using conventional complex wave retrieval method (a) intensity and (b) phase.

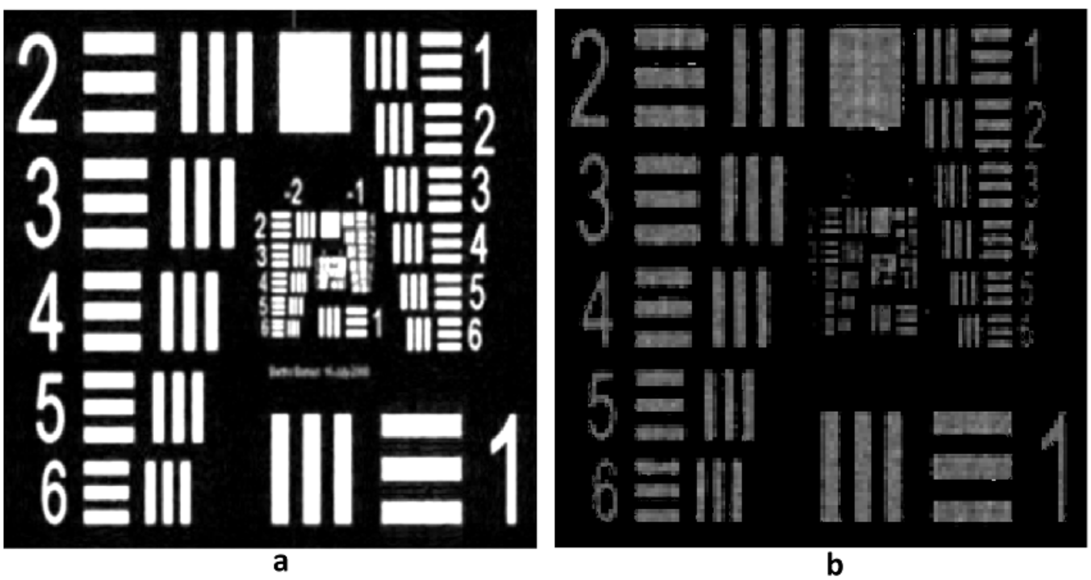

Fig. 9. Complex USAF object wave reconstructed using compressive complex wave retrieval method with Haar wavelet sparsification (a) intensity and (b) phase.

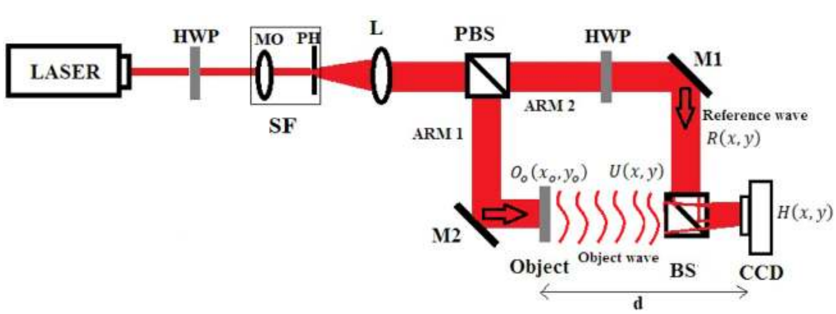

Fig. 10. Schematic of digital holographic interferometric system for the characterization of microlens. HWP-Half Wave Plate ($\lambda/2$), SF-Spatial filter module, MO-Microscope Objective, PH-Pin Hole, L-Collimating Lens, BS-Beam Splitter, M-Mirror, ARM1-Object wave, ARM2-Reference wave.

wave retrieval technique. Since the diameter of the individual lens in the array used in this experiment was comparatively large (∼2mm), a microscope objective was not used for imaging the details of the specimen. The experimental set-up can be modified for imaging in the microscopic regime by choosing a suitable microscope objective and can be accounted for the computational reconstruction algorithm. The 2D CMOS sensor was interfaced with the computer for further numerical computations of the detected digital hologram.

Table 1
Deviation (norm) between original and reconstructed complex object waves.

Methods	Input complex grating object wave		Input complex USAF object wave	
	Intensity deviation	Phase deviation	Intensity deviation	Phase deviation
Conventional complex wave retrieval algorithm	11.7736	56.9695	57.21	578.64
Compressive complex wave retrieval method with Haar wavelet sparsification	11.3065	2.0081	55.62	59.82

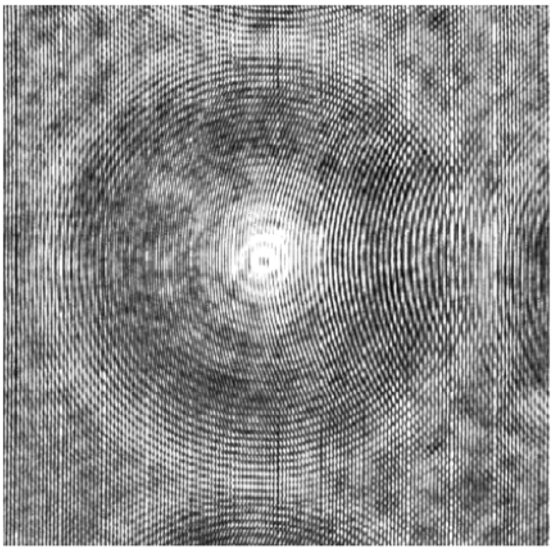

Fig. 11. Digitally recorded off-axis Fresnel hologram of a microlens.

6.1. Experimental results and discussion

A digital hologram of size 512 × 512 pixels from the detected off-axis digital hologram has been considered for the experimental demonstration of the proposed compressive complex object retrieval algorithm and is shown in Fig. 11. The Haar wavelet sparsification in the object field was done as specified by Eq. (8). The numerical reconstruction was done using mathematical operations described in Sections 2 and 3. Fig. 12(a) and 12(b) show the reconstructed intensity and phase respectively of the microlens using the conventional method *i.e.* complex wave retrieval without using CS algorithm. It can be observed from Fig. 12 that there is deformation due to loss of information and presence of noise in both reconstructed intensity and phase of the microlens sample and thus this information is incomplete by nature. These effects will degrade the quality of characterization of the microlens under study. Using the CS optimization given in Eq. (8), an accurate complex object wave of the microlens sample was searched from the incomplete measurement $\tilde{U}(x, y)$ obtained by the conventional method. The results obtained are shown in Fig. 13. Fig. 13(a) shows the reconstructed intensity of the microlens and the noise present in the proposed CS method was very minimal when compared to that of conventional method. It can be inferred from Fig. 13(b) that the reconstructed quality of the microlens phase has significantly improved when compressive complex wave retrieval algorithm with Haar wavelet sparsification of the object field was used. Therefore, the high-quality image reconstruction in quantitative phase imaging is possible with proposed CS method from a single off-axis digital Fresnel hologram.

Fig. 14 shows the comparison of plots of the wrapped phase profile of the microlens using both conventional complex wave retrieval and compressive complex wave retrieval methods. The wrapped phase profile was plotted along the centre of the microlens in both horizontal and vertical directions as shown in Fig. 14(a) and 14(b) respectively. The comparison of results presented in Fig. 14 have revealed that the compressive complex wave retrieval method (CS method) with Haar wavelet sparsification has minimized the noise present in the phase distribution and hence improved the wrapped phase profile of the microlens.

6.2. Characterization of the microlens

The experimental characterization of the microlens was carried out as explained in Section 4. Fig. 15(a) and 15(b) show the unwrapped phase of the microlens obtained by conventional method and proposed CS method respectively. By comparing the results shown in Fig. 15(a) and 15(b), it can be observed that the phase map of the microlens obtained using compressive complex wave retrieval method was better than that of the conventional method.

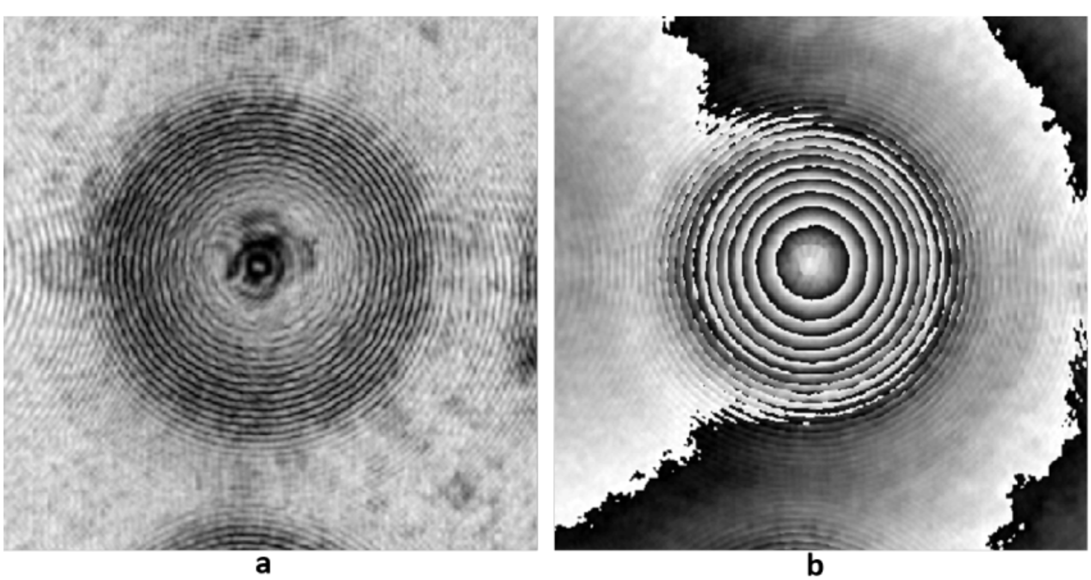

Fig. 12. Reconstructed results using conventional complex wave retrieval method (without CS method) (a) intensity and (b) corresponding phase of the microlens.

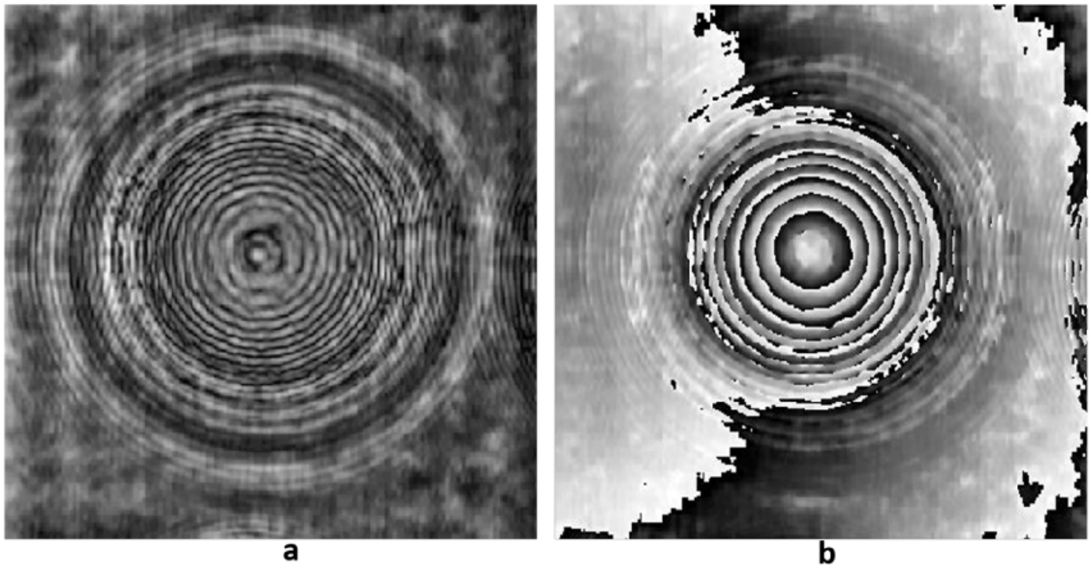

Fig. 13. Reconstructed results using compressive complex wave retrieval method with Haar wavelet sparsification (a) intensity and (b) corresponding phase of a microlens.

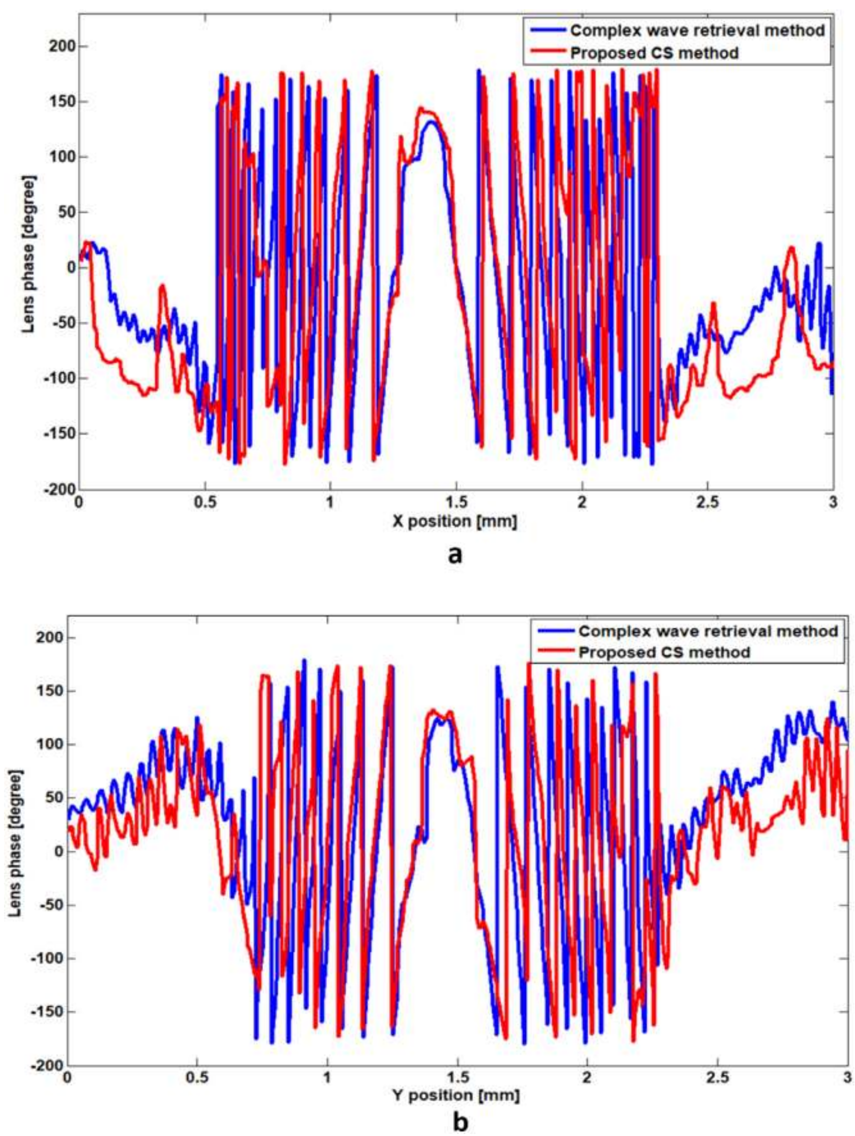

Fig. 14. Comparison of reconstructed phase profile using conventional complex wave retrieval method and proposed compressive complex wave retrieval method with Haar wavelet sparsification (a) wrapped phase profile in horizontal direction (b) wrapped phase profile in vertical direction.

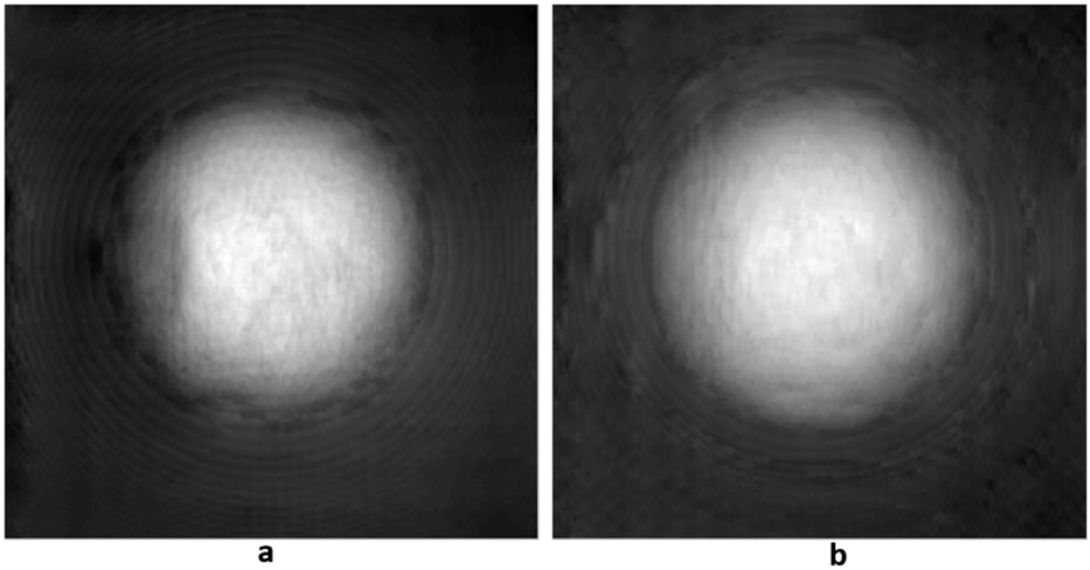

Fig. 15. Unwrapped phase of the microlens (a) conventional complex wave retrieval (without CS method) and (b) Proposed compressive complex wave retrieval method with Haar wavelet sparsification.

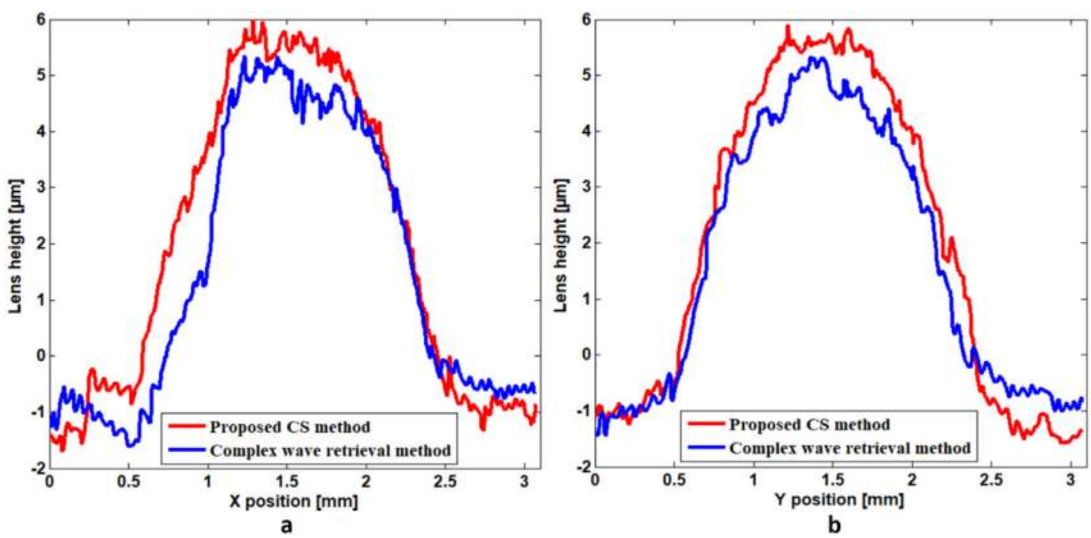

Fig. 16. Comparison of microlens surface profile using conventional complex wave retrieval and proposed CS model with Haar wavelet sparsification (a) horizontal direction (b) vertical direction.

The height profile and diameter of the microlens were calculated for both the methods using Eq. (14) and is shown in Fig. 16. It can be seen from the plots that the results obtained by compressive complex wave retrieval method stands superior to that of the conventional complex wave retrieval method. The experimentally obtained values of h, D and ROC of the microlens are presented in the Table 2 for both the reconstruction methods. Fig. 17(a) and 17(b) show the 3D height map of the microlens obtained using conventional and compressive complex wave retrieval methods respectively. The results show that in compressive complex wave retrieval method with Haar wavelet sparsification, the distortions were minimum and also it has improved the diameter and height profile of the microlens in close agreement with the specifications.

The reconstruction algorithm is executed using different types of input complex object functions in the PC platform (64-bit Intel Core i5-7200U, 2.71 GHz, 8 GB memory) and with MATLAB software (version R2014a). Table 3 shows that the reconstruction time for all the computations using the GPSR algorithm.

7. Conclusions

The compressive digital holography performs well when the underlying holographic scheme is linear, as CS requires a linear sensing process. The strength of digital holography lies in its ability to reconstruct an accurate and quantitative phase of the object along with its intensity information. In order to adapt holography with CS frame work, it has been a convention to discard the nonlinear terms in the holographic equations as model error. This assumption will lead to a deviation in the actual phase measurement, when experimentally observed digital holograms are reconstructed using CS frame work. In this paper, we have demonstrated a compressive sensing (CS) approach for the complex wave retrieval algorithm from single off-axis digital Fresnel holography to improve the accuracy of quantitative phase measurement. The linear model of the non-linear holographic process used in the complex wave retrieval algorithm [40] was utilized to meet the linearity requirement of the proposed CS implementation discussed in this paper. The linearity in the complex object wave retrieval process

Table 2
Measurement of h, D and ROC of the microlens using conventional complex wave retrieval method and the proposed compressive complex wave retrieval method.

Microlens characterization	Conventional method			Proposed CS method		
	D (mm)	h (μm)	ROC (mm)	D (mm)	h (μm)	ROC (mm)
X-Position (horizontal direction)	1.7580	5.3	72.8933	1.8600	5.8	74.5632
Y-Position (vertical direction)	1.8040	5.3	76.7577	1.9380	5.9	79.5729

Table 3
CPU time for the complex object wave reconstruction using GPSR algorithm.

Type of input complex object wave of image size (512 × 512) pixels dimension	CPU time for 20 Iterations (s)
Complex grating image	629.4
Complex USAF chart image	791.8
Microlens array	695.3

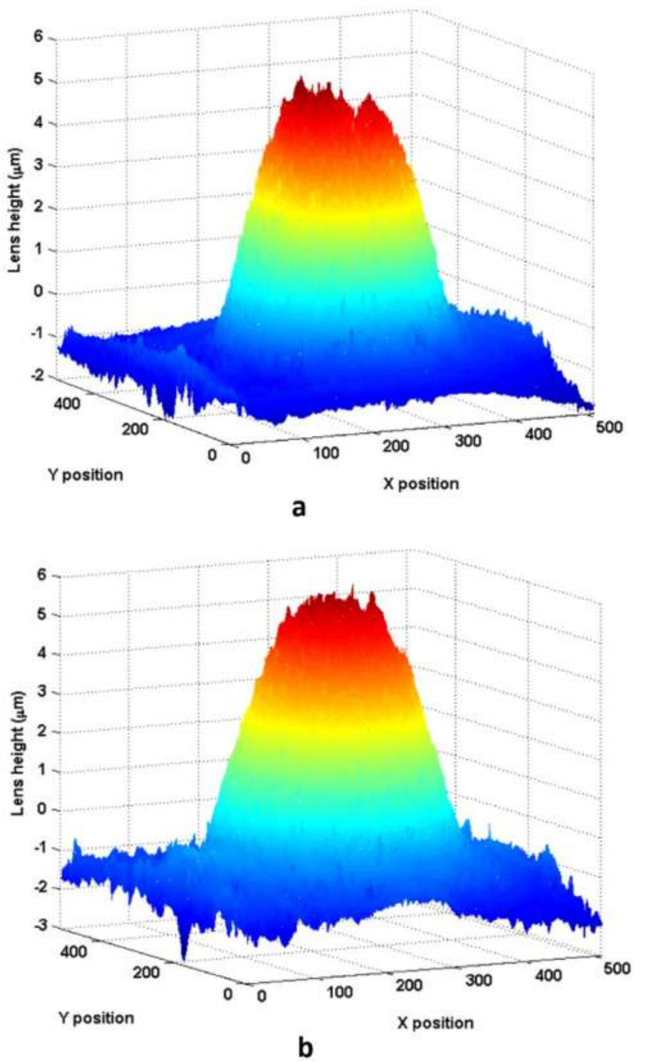

Fig. 17. 3D height map of the microlens (a) reconstructed using conventional complex wave retrieval method and (b) proposed compressive complex wave retrieval method.

from single off-axis digital Fresnel hologram is primarily achieved by a practical assumption that the amplitude of the reference wave is greater than that of the object wave. This assumption is important not only in making the approximation of a non-linear holographic formulation to a linear process but also that, it can be well addressed in an experiment. The approximations used in complex wave retrieval algorithm have resulted in an error in the object field reconstruction, which was compensated by combining it with proposed CS approach. The quality of the object phase retrieved using the proposed method was found to be superior than that of the conventional method. The results were verified and compared using both computational and experimental means. Thus, for actual object phase search using CS frame work, the single shot off-axis digital Fresnel holography scheme with complex wave retrieval method becomes a good choice to maintain the linearity requirement. The phase shifting digital holographic scheme could have been an exact alternative solution to implement CS algorithm, but, the recording of multiple holograms with reference wave phase shifts would limit the dynamic sensing of events and it also demands large data handling.

The experimental demonstration of the compressive complex wave retrieval from single off-axis digital hologram was used for the characterization of the microlens object sample by considering the parameters such as height, diameter and radius of curvature. In order to meet the sparsity criteria in the experiment, Haar wavelet was used as a sparsifier. On comparing the experimental results of compressive complex wave retrieval method with conventional method, it was observed that the compressive complex wave retrieval is superior in reconstruction accuracy with respect to both intensity and quantitate phase information of the object field. The successful implementation of the proposed CS adapted single shot complex wave retrieval method suggests that it can be a good tool for quantitative phase contrast imaging and observing the dynamic events of macroscopic or microscopic objects.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] S. Boyd, L. Vandenberghe, Convex Optimization, Cambridge University Press, 2004, http://dx.doi.org/10.1017/cbo9780511804441.

[2] D.L. Donoho, Compressed sensing, IEEE Trans. Inform. 52 (2006) 1289–1306, http://dx.doi.org/10.1109/TIT.2006.871582.

[3] M.A.T. Figueiredo, R.D. Nowak, S.J. Wright, Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems, IEEE J. Sel. Top. Signal Process. 1 (2007) 586–597, http://dx.doi.org/10.1109/JSTSP.2007.910281.

[4] M. Lustig, D. Donoho, Compressed sensing MRI, IEEE Signal Process. Mag. 25 (2008) 72–82, http://dx.doi.org/10.1109/MSP.2007.914728.

[5] T. v. Sreenivas, W.B. Kleijn, Compressive sensing for sparsely excited speech signals, in: ICASSP, IEEE International Conference on Acoustics, Speech and Signal Processing - Proceedings, 2009, pp. 4125–4128, http://dx.doi.org/10.1109/ICASSP.2009.4960536.

[6] J. Ke, A. Ashok, M.A. Neifeld, Block-wise motion detection using compressive imaging system, Opt. Commun. 284 (2011) 1170–1180, http://dx.doi.org/10.1016/j.optcom.2010.11.028.
[7] D.J. Brady, K. Choi, D.L. Marks, R. Horisaki, S. Lim, Compressive holography, Opt. Express 17 (2009) 13040–13049, http://dx.doi.org/10.1364/oe.17.013040.
[8] Y. Rivenson, A. Stern, B. Javidi, Improved depth resolution by single-exposure in-line compressive holography, Appl. Opt. 52 (2013) A223–31, http://dx.doi.org/10.1364/AO.52.00A223.
[9] Y. Rivenson, A. Stern, B. Javidi, Overview of compressive sensing techniques applied in holography [Invited], Appl. Opt. 52 (2013) A423, http://dx.doi.org/10.1364/AO.52.00A423.
[10] M.S. Mozhou Sha, J.L. Juan Liu, X.L. Xin Li, Y.W. Yongtian Wang, Holographic display based on compressive sensing (Invited Paper), Chin. Opt. Lett. 12 (2014) 060023, http://dx.doi.org/10.3788/col201412.060023.
[11] P. Clemente, V. Durán, E. Tajahuerce, P. Andrés, V. Climent, J. Lancis, Compressive holography with a single-pixel detector, Opt. Lett. 38 (2013) 2524–2527, http://dx.doi.org/10.1364/ol.38.002524.
[12] Y. Rivenson, A. Stern, B. Javidi, Compressive Fresnel holography, IEEE/OSA J. Display Technol. 6 (2010) 506–509, http://dx.doi.org/10.1109/JDT.2010.2042276.
[13] A. Ashok, M.A. Neifeld, Compressive imaging: hybrid measurement basis design, J. Opt. Soc. Amer. A 28 (2011) 1041–1050, http://dx.doi.org/10.1364/josaa.28.001041.
[14] P. Ramachandran, Z.C. Alex, A. Nelleri, Compressive Fresnel digital holography using Fresnelet based sparse representation, Opt. Commun. 340 (2015) 110–115, http://dx.doi.org/10.1016/j.optcom.2014.11.043.
[15] P. Ramachandran, Z.C. Alex, A. Nelleri, Phase reconstruction using compressive two-step parallel phase-shifting digital holography, Opt. Eng. 57 (2018) 1, http://dx.doi.org/10.1117/1.oe.57.4.043105.
[16] X. Wu, Y. Yu, W. Zhou, A. Asundi, 4f amplified in-line compressive holography, Opt. Express 22 (2014) 19860, http://dx.doi.org/10.1364/OE.22.019860.
[17] Y. Endo, T. Shimobaba, T. Kakue, T. Ito, GPU-accelerated compressive holography, Opt. Express 24 (2016) 8437–8445, http://dx.doi.org/10.1364/oe.24.008437.
[18] T. Leportier, M.C. Park, Holographic reconstruction by compressive sensing, J. Opt. (United Kingdom) 19 (2017) 6, http://dx.doi.org/10.1088/2040-8986/aa6b62.
[19] M.M. Marim, M. Atlan, E. Angelini, J.-C. Olivo-Marin, Compressed sensing with off-axis frequency-shifting holography, Opt. Lett. 35 (2010) 871–873, http://dx.doi.org/10.1364/ol.35.000871.
[20] P. Memmolo, M. Paturzo, A. Pelagotti, A. Finizio, P. Ferraro, B. Javidi, Compression of digital holograms via adaptive-sparse representation, Opt. Lett. 35 (2010) 3883–3885, http://dx.doi.org/10.1364/ol.35.003883.
[21] P. Memmolo, I. Esnaola, A. Finizio, M. Paturzo, P. Ferraro, A.M. Tulino, SPADEDH: a sparsity-based denoising method of digital holograms without knowing the noise statistics, Opt. Express 20 (2012) 17250–17257, http://dx.doi.org/10.1364/oe.20.017250.
[22] X.C. Xuemei Cao, X.S. Xinzhu Sang, Z.C. Zhidong Chen, Y.Z. Ying Zhang, J.L. Junmin Leng, N.G. Nan Guo, B.Y. Binbin Yan, J.Y. Jinhui Yuan, K.W. Kuiru Wang, C.Y. Chongxiu Yu, Fresnel hologram reconstruction of complex three-dimensional object based on compressive sensing, Chin. Opt. Lett. 12 (2014) 080901, http://dx.doi.org/10.3788/col201412.080901.
[23] P. Memmolo, M. Paturzo, B. Javidi, P.A. Netti, P. Ferraro, Refocusing criterion via sparsity measurements in digital holography, Opt. Lett. 39 (2014) 4719–4722, http://dx.doi.org/10.1364/ol.39.004719.
[24] F. Charrière, J. Kühn, T. Colomb, F. Montfort, E. Cuche, Y. Emery, K. Weible, P. Marquet, C. Depeursinge, Characterization of microlenses by digital holographic microscopy, Appl. Opt. 45 (2006) 829–835, http://dx.doi.org/10.1364/AO.45.000829.
[25] Q. Weijuan, C.O. Choo, Y. Yingjie, A. Asundi, Microlens characterization by digital holographic microscopy with physical spherical phase compensation, Appl. Opt. 49 (2010) 6448–6454, http://dx.doi.org/10.1364/AO.49.006448.
[26] S. Reichelt, H. Zappe, Combined Twyman-Green and Mach–Zehnder interferometer for microlens testing, in: Appl. Opt., 2005, pp. 5786–5792, http://dx.doi.org/10.1364/AO.44.005786.
[27] K. Khare, P.T.S. Ali, J. Joseph, Single shot high resolution digital holography, Opt. Express 21 (2013) 2581–2591, http://dx.doi.org/10.1364/oe.21.002581.
[28] A.K. Agarwal, A. Aran, U. Gopinathan, A.N. Kaul, A.K. Gupta, Fabrication, replication and characterization of microlens arrays for imaging applications, in: 2012 International Conference on Fiber Optics and Photonics, PHOTONICS 2012, Optical Society of America, 2012, p. M1B.3, http://dx.doi.org/10.1364/photonics.2012.m1b.3.
[29] L. Miccio, A. Finizio, S. Grilli, V. Vespini, M. Paturzo, S. de Nicola, P. Ferraro, Tunable liquid microlens arrays in electrode-less configuration and their accurate characterization by interference microscopy, Opt. Express 17 (2009) 2487–2499, http://dx.doi.org/10.1364/oe.17.002487.
[30] S. Grilli, L. Miccio, V. Vespini, A. Finizio, S. de Nicola, P. Ferraro, Liquid microlens array activated by selective electrowetting on lithium niobate substrates, Opt. Express 16 (2008) 8084–8093, http://dx.doi.org/10.1364/oe.16.008084.
[31] V. Vespini, S. Coppola, M. Todino, M. Paturzo, V. Bianco, S. Grilli, P. Ferraro, Forward electrohydrodynamic inkjet printing of optical microlenses on microfluidic devices, Lab on a Chip (2016) http://dx.doi.org/10.1039/c5lc01386k.
[32] L. Miccio, D. Alfieri, S. Grilli, P. Ferraro, A. Finizio, L. de Petrocellis, S.D. Nicola, Direct full compensation of the aberrations in quantitative phase microscopy of thin objects by a single digital hologram, Appl. Phys. Lett. 90 (2007) 041104, http://dx.doi.org/10.1063/1.2432287.
[33] U. Schnars, W. Jüptner, Direct recording of holograms by a CCD target and numerical reconstruction, Appl. Opt. 33 (1994) 179–181, http://dx.doi.org/10.1364/AO.33.000179.
[34] J.W. Goodman, Introduction To Fourier Optics, second ed., Optical Engineering, 1996, http://dx.doi.org/10.1117/1.601121.
[35] U. Schnars, W. Jueptner, Digital Holography: Digital Hologram Recording, Numerical Reconstruction, and Related Techniques, Springer, 2005, http://dx.doi.org/10.1007/b138284.
[36] P. Picart, J. Leval, General theoretical formulation of image formation in digital Fresnel holography, J. Opt. Soc. Amer. A 25 (2008) 1744–1761, http://dx.doi.org/10.1364/josaa.25.001744.
[37] Y. Frauel, T.J. Naughton, O. Matoba, E. Tajahuerce, B. Javidi, Three-dimensional imaging and processing using computational holographic imaging, Proc. IEEE (2006) http://dx.doi.org/10.1109/JPROC.2006.870704.
[38] I. Yamaguchi, T. Zhang, Phase-shifting digital holography, Opt. Lett. 22 (1997) 1268–1270, http://dx.doi.org/10.1364/ol.22.001268.
[39] M.K. Kim, Phase-shifting digital holography, Springer Ser. Optical Sci. 162 (2011) 95–108, http://dx.doi.org/10.1007/978-1-4419-7793-9_8.
[40] M. Liebling, T. Blu, M. Unser, Complex-wave retrieval from a single off-axis hologram, J. Opt. Soc. Amer. A 21 (2004) 367–377, http://dx.doi.org/10.1364/josaa.21.000367.
[41] A. Nelleri, U. Gopinathan, J. Joseph, K. Singh, Wavelet based three-dimensional object recognition using single off-axis digital Fresnel hologram, in: Opto-Ireland 2005: Photonic Engineering, 2005, p. 30, http://dx.doi.org/10.1117/12.605085.
[42] A. Nelleri, Digital Fresnel field encryption for three-dimensional information security, Opt. Eng. 46 (2007) 045801, http://dx.doi.org/10.1117/1.2727377.
[43] A. Nelleri, U. Gopinathan, J. Joseph, K. Singh, Three-dimensional object recognition from digital Fresnel hologram by wavelet matched filtering, Opt. Commun. 259 (2006) 499–506, http://dx.doi.org/10.1016/j.optcom.2005.09.007.
[44] A. Nelleri, J. Joseph, K. Singh, Recognition and classification of three-dimensional phase objects by digital Fresnel holography, Appl. Opt. 45 (2006) 4046–4053, http://dx.doi.org/10.1364/AO.45.004046.
[45] D.C. Ghiglia, L.A. Romero, Robust two-dimensional weighted and unweighted phase unwrapping that uses fast transforms and iterative methods, J. Opt. Soc. Amer. A 11 (1994) 107–117, http://dx.doi.org/10.1364/josaa.11.000107.