Kaushik Dey
Student
School of Mechanical Engineering
VIT University
India

Jay Rohit Verlekar
Student
School of Mechanical Engineering
VIT University
India

Sourabh Kumar Soni
Research Scholar
School of Mechanical Engineering
VIT University
India

Benedict Thomas
Associate Professor
School of Mechanical Engineering
VIT University
India

Dynamic Analysis of Functionally Graded Shaft

This paper deals with modal analysis of a functionally graded composite rotor shaft. Material properties distribution is assumed to vary along the radial direction of the Functionally Graded shaft according to power law distribution. The rotor shaft has been modelled and modal analysis is carried out. Functionally Graded Rotor shaft is analyzed by applying the loads and boundary conditions to obtain the vibration response and Campbell diagram is plotted. Various results have been obtained such as Campbell diagram, stability speed limit and time responses for functionally graded shaft and also compared with conventional steel shaft. It has been found that the responses of the functionally graded shaft are significantly influenced by material properties, radial thickness and power law index. The results obtained also show the benefits of functionally graded shaft over conventional steel shaft.

Keywords: *Modal Analysis, Power Law, Functionally Graded materials, Rotor Shaft, Campbell Diagram.*

1. INTRODUCTION

Vibrational losses in machines are unavoidable where there are rotating parts involved. It is important to have a clear knowledge about modal frequencies of rotating systems to avoid resonance phenomenon leading to complete breakdown. Special precautions need to be taken even while crossing these modal frequencies. For example, if first modal frequency is 1550 rpm at which resonance occurs and we need to attain a frequency of 2000 rpm we need to add vibration arrestors to go through the first modal frequency. Another important aspect to reduce the vibrational losses in machines is the use of more effective materials.

Composites have a diverse range of applications in mechanical structures owing to lower weight to strength ratio. Weakness at the interface between adjacent layers is one of the major drawbacks in composites; this phenomenon is commonly known as delamination and this is the main reason behind breakdown of machinery. To deal with this shortcoming, a different type of material was brought forth, namely Functionally Graded Materials (FGMs). FGMs are those materials whose characteristics vary with respect to certain directions and parameters and this is how it removes the problem of weakness at interfaces. In FGMs, volume proportions of two or more than two materials are changed gradually with respect to specific directions to obtain desired properties. This gradual change allowed for a more uniform stress distribution in the component, and as a result, they improved in quality and durability. Use of FGMs results in smart resistance and better performance in extreme conditions such as those involving high pressure and high temperature. FGMs exhibit higher strength, lower density and higher stiffness and material characteristics than their metallic counterparts; this fact led to the concept of replacing metallic shafts with FG shafts in many fields like aerospace, automobile, defence, energy, marine, opto-electronics, and thermo-electronics; in machines like turbine shafts, thermo-electric generators and other manufacturing/machining equipment's [1]. In FGMs the local physical and chemical properties of materials are different, with the changes in properties taking place along certain predetermined directions. Abrupt changes in material properties, which is the phenomenon taking place in composites, can be eliminated by using power law gradation in FGMs; this gradation would prevent delamination phenomenon, reduce stress between laminates and give better strength within material.

Many basic machine parts are designed using FGMs. Machine parts are more likely to fail by large deflections due to combined effect of thermo mechanical loading. For such boundary conditions FGMs are used in high temperature variations along the length of the material. FGMs are mostly man-made from isotropic materials like metals and ceramics. A comparative study was done by Sino et al. [2] between the proposed Simplified Homogenized Beam Theory (SHBT) and the existing EMBT, and the former is shown to be superior to the latter in that the SHBT considers the distance between composite layers and neutral axis. In addition to this, it also considers the internal damping by using the specific damping capacity of each ply of the composite assembly, and takes into account the transversal flexural shear. Hosseini et al. [3] analyzed the combination resonance in combination mode of a simple rotating shaft by using harmonic balancing method, and they studied the effects of, eccentricity, diametrical mass of inertia and external damping on the steady state response of the rotating

Received: July 2018, Accepted: November 2018
Correspondence to: Dr. Benedict Thomas,
School of Mechanical Engineering,
VIT University, Vellore, 632014, India
E-mail: benedict.thomas@vit.ac.in
Doi: 10.5937/fmet1901151D
© Faculty of Mechanical Engineering, Belgrade. All rights reserved

shaft. Experimental vibrational analysis was carried out by Jain et al. [4] on multi-cracked rotor shafts. Variation in stiffness of material was observed with variation in crack depth. Stiffness variation was observed in the presence of second crack as well. Nonlinear dynamic behavior was studied while considering effects of crack location and shaft speed too. Khadem et al. [5] studied and analyzed a geometrically non-linear inextensible rotor shaft to obtain critical speeds by applying method of complex differential equation. It was shown that the results obtained, like the frequency curves and amplitude diagrams, were the same from application of the two approaches. Dasgupta and Rajan [6] analyses the dynamic responses of a compound rotor system subjected to the external and internal damping.

Montagnier et al. [7] investigated the composite shaft to obtain instabilities like unstable whirl oscillations triggered by damping in the supercritical regime. As a result, they were able to obtain the whirling speeds of the supercritical shaft mounted on elastic supports in the analytical form. It was found that shafts having $\pm 30^0$ or $\pm 45^0$ orientation of plies cannot be used in supercritical zone because of extremely high losses. Gayen et al. [8], through FE formulations, determined the effects of temperature variation, crack dimensions, power law gradient index, slenderness ratio on the dynamic characteristics of the FG rotating shaft system. They found that the whirl frequencies vary significantly with the gradient index for cracked and uncracked FG rotor shaft systems. Further they calculated the Local Flexibility Coefficients (LFCs) for varying crack depths and varying crack closer lines for a cracked FG shaft and they studied the effect of crack depth and crack closer line position on the LFCs. They concluded that, for a particular crack configuration, LFCs values attain their maximum for metallic shafts and minimum for ceramic shafts. Furthermore, it was found that LFCs values increase with the increasing power law gradient indices [9]. Liu et al. [10] studied the un-damped vibration of exponential FG beams with only one delamination, by developing an analytical solution for the same. They concluded that shifting of neutral axis should be taken into account while formulating the continuity conditions for delaminated beam, because of asymmetrical distribution of material property in FGMs. Badour et al. [11] on realizing that the rapid and short period Fourier transforms could not efficiently analyze non-stationary signals, considered wavelet transform tools to mitigate this situation. In order to analyze the monitored vibration signals, two popular wavelet methods, namely continuous wavelet and wavelet packet transforms were used. Moreover, a novel algorithm, which incorporates a combination of both these and the concept of windowing a particular pulse into several revolutions of shafts to emphasize faults, was proposed and implemented in this paper. Koizumi [12] discussed and introduced functionally graded material. Rao et al. [13] came up with the idea of a mathematical model using power law and TMBT for FE modelling. Boukhalfa et al. [14] analyzed FG rotor shaft by using P-version of FEM with trigonometric shape functions and studied whirling frequencies, natural frequency response, and the whirling speeds of the rotating FG shaft. Further they also employ hp-version of FEM in to study vibrations of composite shaft by using kinetic and strain energy equations to formulate the equation of motion [15]. In the available literature very few research work reported on the modal analysis of FGM rotor shaft. The present work intends to demonstrate the superiority of FG shaft over steel shaft via Finite Element Analysis on ANSYS. It also provides an insight on influence of material properties, radial thickness and power law index on vibration response of FG shaft.

2. MODELLING AND ANALYSIS

In the present work the FG shaft is modelled using ANSYS considering rotation in x direction (axis of rotation of shaft) is free and is zero in the y and z directions. The shaft is modelled as a shell having mean diameter 129.5 mm as shown in Fig. 1 and other parameters as mentioned in table 1 and 2.

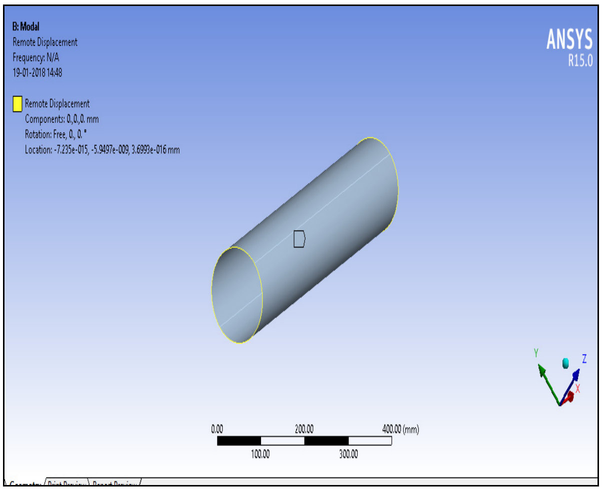

Figure 1. Rotor Shaft Model

In order to achieve the best performance of Functionally Graded rotating shafts, which are heterogeneous, precisely estimated material property and optimal volume fraction selection are necessary.

Table 1. Shaft parameters

Shaft parameter	Parameter value
Length	0.720 m
Thickness	0.01 m
Mass	22.994 kg

Table 2. Range of rotational velocities (from points 1 to 7)

Points	X (rad/s)
1	52.38
2	209.52
3	419.05
4	628.57
5	838.1
6	1047.6
7	1257.1

An FG shaft as shown in Figure 2 has been considered, having finite length L, inner radius (r_a) and outer radius (r_b). And it has taken Aluminium and Alumina as top and bottom surfaces respectively.

2.1 Modelling of shaft

After taking into consideration the inertia and gyros–copic couple effects the FG shaft is modelled using ANSYS solver. The shaft is modelled as a shell having mean diameter 129.5 mm and it undergoes rotation for a range of angular velocity about its geometrical axis. The x axis is assumed to be the shaft geometrical axis. The displacement fields are given as follows:

$$\begin{aligned} u_x(x,y,z,t) &= z\beta_x(x,t) - y\beta_y(x,t) \\ v_y(x,y,z,t) &= v_0(x,t) \\ w_z(x,y,z,t) &= w_0(x,t) \end{aligned} \tag{1}$$

By utilizing the kinetic energy and the elastic potential energy of the FG shaft, external loads and bearings performing virtual work done, combined with Hamilton's principle for rotating shaft gives rise to the following quations:

$$[A]\{\ddot{x}\} + ([B] + \lambda[C])\{\dot{x}\} + [D]\{x\} = \{E\} \tag{1}$$

In the above equation [A] refers to mass matrix, [B] refers to gyroscopic matrix, [C] refers to damping mat–rix, and [D] refers to the stiffness matrix. Also, in the above equation, {E} is the external force vector and {x} is the displacement vector. From the rotor dynamic Lagrangian equation of motion, including both internal viscous and hysteretic damping of a shaft disk element, the equation can be modified to obtain:

$$\begin{aligned} &[A]\{\ddot{x}\} + \left([B] + \lambda[C] + \eta_v[D]\right)\{x\} + \\ &\left[\left(\frac{1+\eta_H}{\sqrt{1+\eta_H^2}}\right)[D] + \right. \\ &\left.\left[\left(\eta_v\lambda + \frac{\eta_H}{\sqrt{1+\eta_H^2}}\right)[D_{Cir}]\right]\right]\{\dot{x}\} = \{E\} \end{aligned} \tag{3}$$

The variation of properties of Young's modulus, density and Poisson's ratio of the FG shaft to be modelled was obtained using power law, with properties varying along the radial direction and value of k=0.5. The effective material properties P can be written as,

$$P = P_A V_A + P_a V_a \tag{4}$$

Table 3: Material properties of Aluminium and Alumina (Al_2O_3)

Material Properties	Aluminum
Young's Modulus(GPa)	70
Density Kg/m^3	2700
Poisson's ratio	0.35

The relationship between alumina's and aluminum's volume proportions is given by:

$$V_A + V_a = 1 \tag{5}$$

Where V_A and V_a are the volume proportions of alumina and aluminium respectively at any point z throughout the radius r; r_A and r_a are the corresponding radii, where the former is the external radius and the latter is the internal radius of the shaft. According to power law, V_A can be expressed as

$$V_A(z) = \left(\frac{r - r_a}{r_A - r_a}\right)^k \tag{6}$$

$$P(z) = \{P_A(z) - P_a(z)\}V_A(z) + P_a(z) \tag{7}$$

Here, $z = r$.

This particular material distribution was incorpo–rated in the designed shaft of given dimensions. This was achieved by layer wise stacking of different plies of different material composition (as per power law), with the plies stacked in inward direction. Furthermore, the fiber orientation between all the plies was the same.

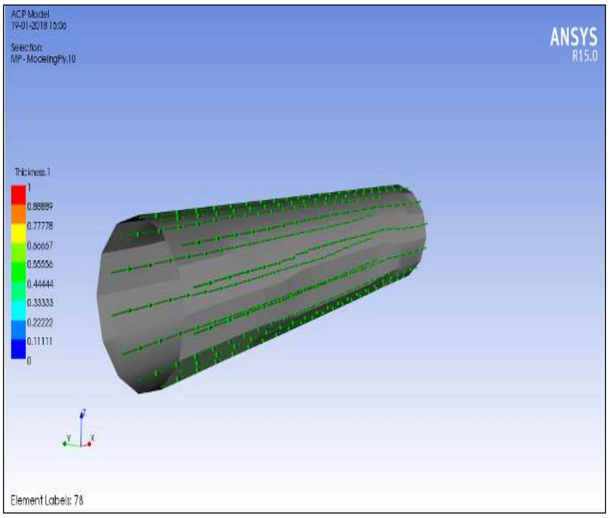

Figure 2. Fibre direction is along reference axis (X axis)

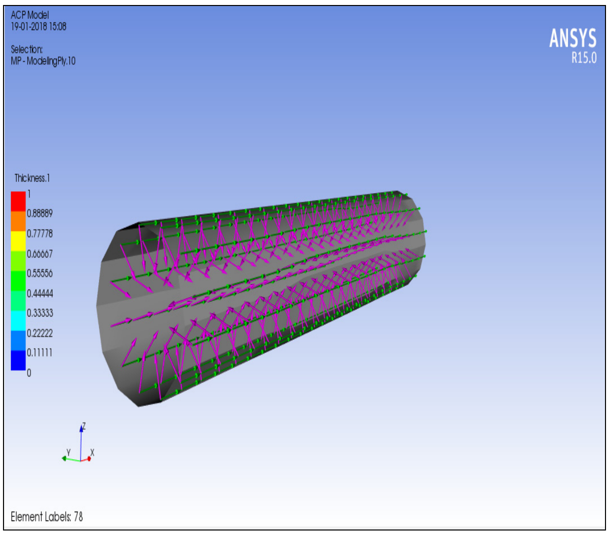

Figure 3. Stacking sequence is in inward direction as Indicated by the radially pointing arrows

2.2 Comparison of whirling speeds through Campbell diagrams

Through the above plot, if observed carefully, it can be seen that the critical speed for the second modal frequ–ency lies in the range of 1100-1200 rad/s. The model of the functionally graded rotor shaft was solved to obtain Campbell Diagram and frequency response:

In the above plot, the critical speed for the second modal frequency occurs at almost 5500 rad/s, which is almost five times the value of the critical speed obtained in the case of conventional steel shaft.

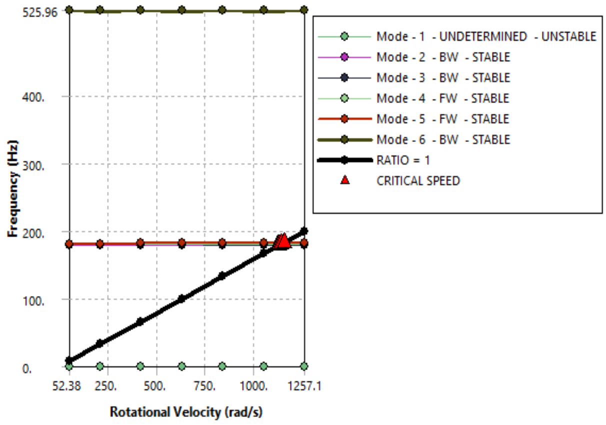

Figure 4. Campbell diagram of conventional steel shaft

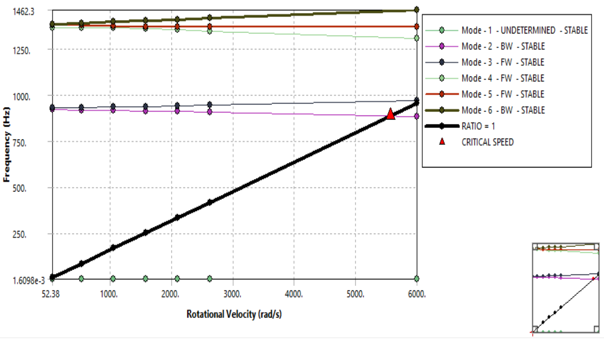

Figure 5. Campbell diagram of FG shaft

Table 4: Modes and their respective frequencies for FG shaft

Mode	Damped Frequency
1	1.6098e-003
2	920.08
3	931.7
4	1366
5	1381.7
6	1386.8

3. MODE SHAPES

After modal analysis mode shapes are plotted for steel shafts and corresponding results are shown in Fig. 6 to Fig 11.

3.1 Comparison of whirling speeds through Campbell diagrams

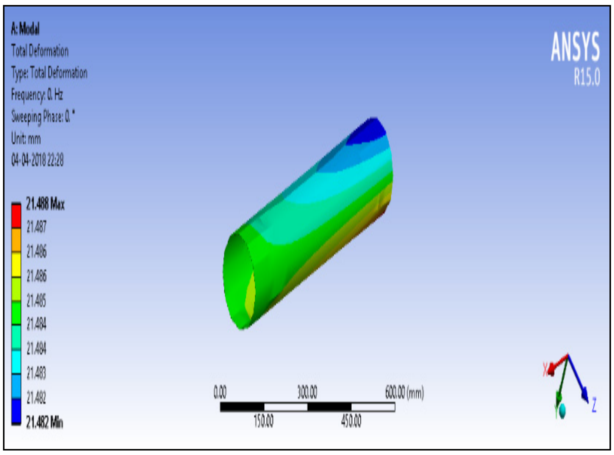

Figure 6. First mode shape - conventional steel shaft

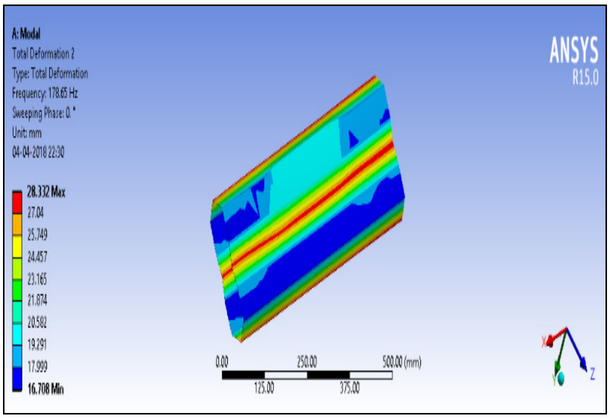

Figure 7. Second mode shape - conventional steel shaft

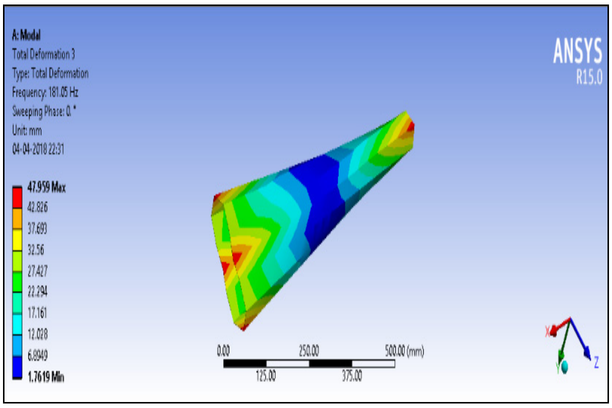

Figure 8. Third mode shape - conventional steel shaft

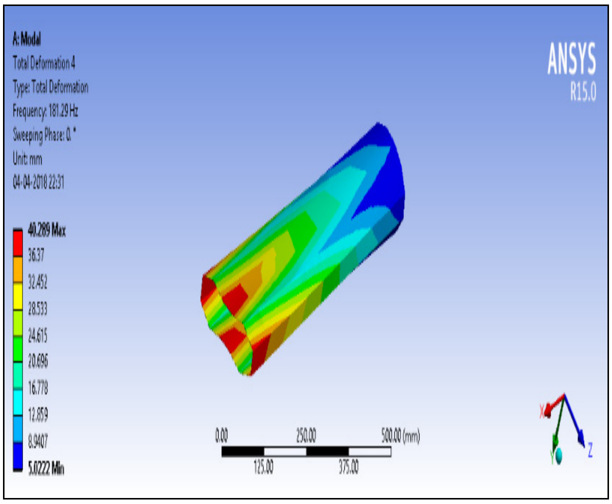

Figure 9. Fourth mode shape- conventional steel shaft

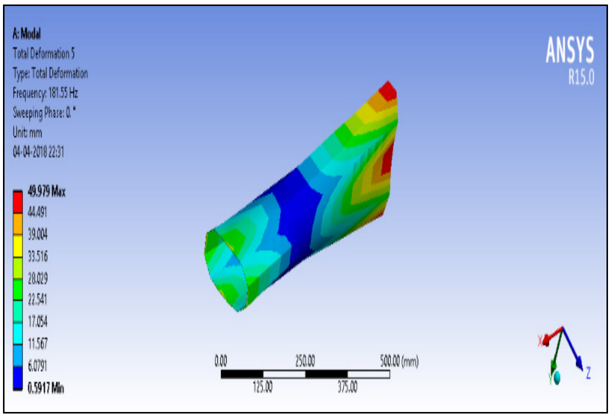

Figure 10. Fifth mode shape- conventional steel shaft

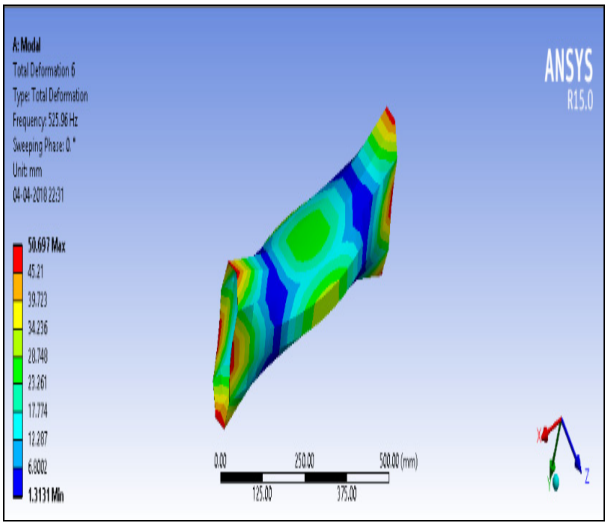

Figure 11. Sixth mode shape- conventional steel shaft

3.2 Mode shapes – FG shaft

After modal analysis mode shapes are plotted for FG shaft and corresponding results are shown in Fig. 12 to Fig 17.

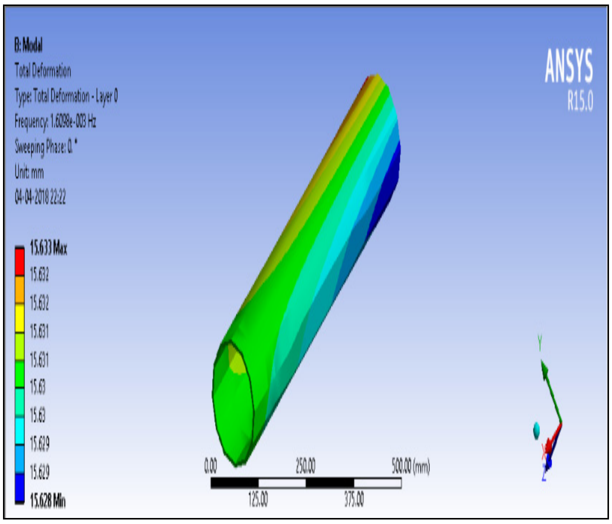

Figure 12. First mode shape- FG shaft

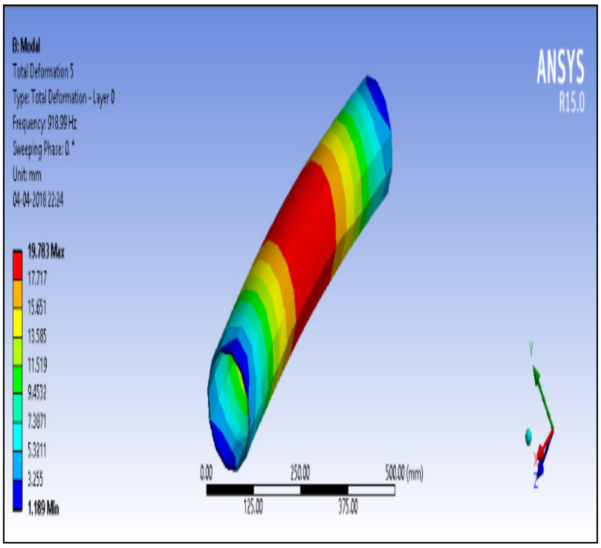

Figure 13. Second mode shape- FG shaft

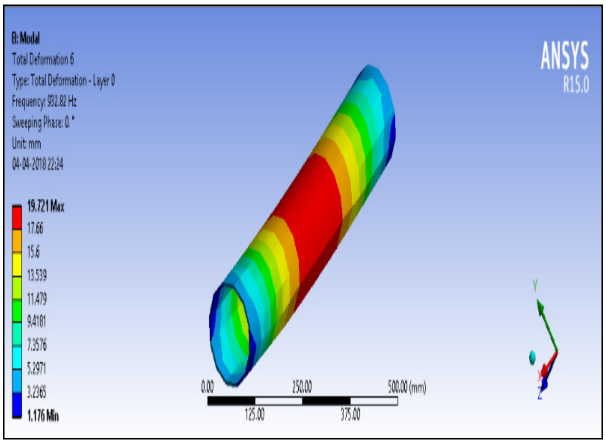

Figure 14. Third mode shape- FG shaft

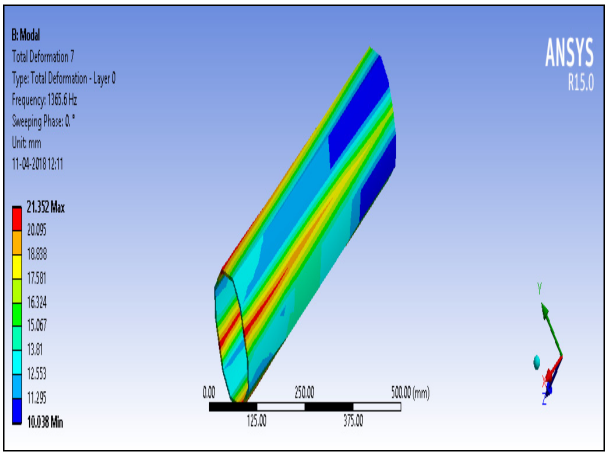

Figure 15. Fourth mode shape-FG shaft

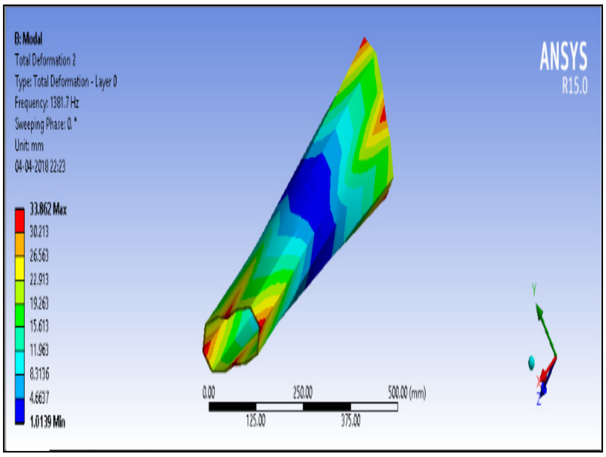

Figure 16. Fifth mode shape- FG shaft

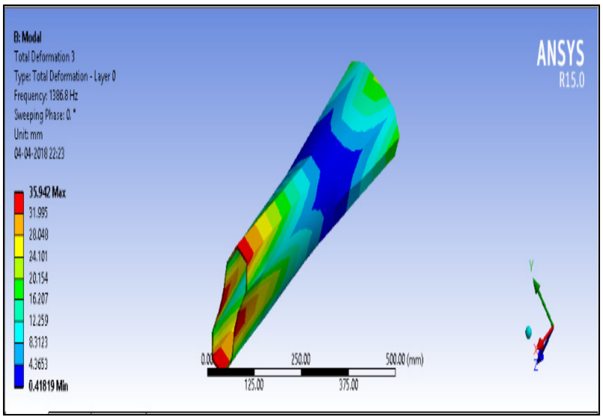

Figure 17. Sixth mode shape- FG shaft

3.3 Comparison of different parameters (shown in the tables below) and inference from the same

Based on the modal analysis comparison of various pa– rameters of steel shaft and FG shaft are presented in table 5 and 6.

Table 5: Comparison of modal frequencies (Hz)

Mode Number	Steel shaft	FG shaft
1	0	1.609e-003
2	178.65	918.99
3	181.05	932.82
4	181.29	1365.6
5	181.55	1381.7
6	525.96	1386.8

Table 6: Comparison of maximum deformation (in mm)

Mode Number	Steel shaft	FG shaft
1	21.488	15.633
2	28.332	19.783
3	47.959	19.721
4	40.289	21.352
5	49.979	33.862
6	50.697	35.942

Table 7: Comparison of minimum deformation (in mm)

Mode	Steel shaft	FG shaft
1	21.482	15.628
2	16.708	1.189
3	1.7619	1.176
4	5.0222	10.038
5	0.5917	1.0139
6	1.3131	0.41819

The results shown in the avbove three tables are the comparison between FG shaft and steel shaft in terms of frequency, maximum deformation and minimum defor– mation. It is evident from the results that FG shaft results are significantly better than the steel shaft.

4. CONCLUSION

It can be observed from the above analysis that Func– tionally Graded shaft is superior to the one made from conventional steel as FGM shaft allows for higher critical speeds, thereby increasing its capability and effectiveness. Also in FG shafts deformations are sma– ller than those in conventional steel shaft for corres– ponding modes. The results obtained show the benefits of functionally graded shaft over conventional steel shaft. Finally it can be concluded that by using rotor shaft made of FG materials shows better dynamic behaviour than conventional steel shaft. The proposed shaft system will be very useful for numerous appli– cations such as driveshaft in automobiles, jet engines and helicopter drive applications, turbines shafts and other rotating machineries.

REFERENCES

[1] Mahamood, R. M., and Akinlabi, E.T.: Types of functionally graded materials and their areas of application, In Functionally Graded Materials, pp. 9-21, Springer, Cham, 2017.

[2] Sino, R., Baranger, T. N., Chatelet, E., and Jacquet, G.: Dynamic analysis of a rotating composite shaft. Composites Science and Technology, Vol. 68, No. 2, pp. 337-345, 2008.

[3] Hosseini, S. A. A., Khadem, S. E.: Combination re– sonances in a rotating shaft. Mechanism and Mac– hine Theory, Vol. 44, No. 8, pp. 1535-1547, 2009.

[4] Jain A. K., Rastogi V., A. K. Agrawal,: Expe– rimental Investigation of Vibration Analysis of Multi-Crack Rotor Shaft: *Proceedings of the 12th International Conference on Vibration Problems, ICOVP 2015*, Vol. 144, pp. 1451–1458, 2016.

[5] Khadem S. E. et al.: Primary resonances of a non– linear in-extensional rotating shaft. Mech. Mach. Theory, Vol. 45, No. 8, pp. 1067–1081, 2010.

[6] Dasgupta, S.S., Rajan, J.A.: Steady-state and tran– sient responses of a flexible eccentric spinning shaft, FME Transactions, Vol. 46, No. 1, pp. 133-137, 2018.

[7] Montagnier O., Hochard C.: Dynamics of a super– critical composite shaft mounted on viscoelastic supports, J. Sound Vib. Vol. 333, No. 2, pp. 470–484, 2014.

[8] Gayen D., Chakraborty D., and R. Tiwari.: Whirl frequencies and critical speeds of a rotor-bearing system with a cracked functionally graded shaft – Finite element analysis, Eur. J. Mech. A/Solids, Vol. 61, pp. 47–58, 2017.

[9] Gayen D. and Chakraborty D.: Variation of Local Flexibility Coefficients of Functionally Graded Cracked Shaft, *Proceedings of the 12th International Conference on Vibration Problems, ICOVP 2015,* Vol. 144, pp. 1443–1450, 2016.

[10] Liu Y., Xiao J., and D. Shu.: Free vibration of exponential functionally graded beams with single delamination, in: *Proceedings of the International Conference on Materials for Advanced Technologies 2013*, Vol. 75, pp. 164–168, 2014.

[11] Al-Badour F. et al.: Vibration analysis of rotating machinery using time- frequency analysis and wavelet techniques, Mech. Syst. Signal Process, Vol. 25, No. 6, pp. 2083–2101, 2011.

[12] Koizumi M.: Concept of FGM, Functionally Gra– dient Materials. Ceramic Trans. 34, pp. 3-10, 1993.

[13] Rao D. K., Roy T.: Vibration Analysis of Functi– onally Graded Rotating Shaft System, in: *Pro– ceedings of the 12th International Conference on Vibration Problems, ICOVP 2015*, Vol. 144, pp. 775–780, 2016.

[14] Boukhalfa A., "Dynamic analysis of a spinning functionally graded material shaft by the p - version of the finite element method," Lat. Am. J. Solids Struct., Vol. 11, No. 11, pp. 2018–2038, 2014.

[15] Boukhalfa A. and Hadjoui A.: Free vibration analysis of an embarked rotating composite shaft using the hp-version of the FEM. Lat. Am. J. Solids Struct., Vol. 7, No. 2, pp. 105–141, 2010.

NOMENCLATURE

V_A	Volume proportions of alumina
V_a	Volume proportions of aluminium

FG	Functionally graded
r_a	Inner radius of the FG shaft
r_b	Outer radius of the FG shaft

ДИНАМИЧКА АНАЛИЗА ФУНКЦИОНАЛНО ГРАДИРАНОГ ВРАТИЛА

К. Дej, J.Р. Верлекар, С.К. Сони, Б. Томас

Рад се бави модалном анализом функционално гра–дираног композитног роторског вратила. Полази се од претпоставке да својства материјала варирају по радијалном правцу вратила према закону о дистри–буцији снаге. Вратило је моделирано и извр–шена је модална анализа. Анализа вратила је обав–љена у условима оптерећења и граничних услова опте–рећења да би се добио вибрацијски одзив и израђен је Кемпбелов дијаграм. Добијени су разли–чити резултати: Кемпбелов дијаграм, граница ста–билне брзине и временски одзив, а извршено је и поређење функционално градираног са конвенци–оналним вратилом од челика. Утврђено је да постоји значајан утицај својстава материјала, радијалне дебљине и индекса закона снаге на функционално градирано вратило. Резултати такође показују пред–ности градираног у односу на конвенционално челично вратило.