PAPER • OPEN ACCESS

Effect of partial slip and chemical reaction on convection of a viscoelastic fluid over a stretching surface with Cattaneo-Christov heat flux model

To cite this article: S Eswaramoorthi *et al* 2017 *IOP Conf. Ser.: Mater. Sci. Eng.* **263** 062009

View the article online for updates and enhancements.

Related content

- Dufour and Soret effects on MHD convection of Oldroyd-B liquid over stretching surface with chemical reaction and radiation using Cattaneo-Christov heat flux
  K. Loganathan, S. Sivasankaran, M. Bhuvaneswari et al.

- Newtonian heating effects in three-dimensional flow of viscoelastic fluid
  A. Qayyum, T. Hayat, M. S. Alhuthali et al.

- Effects of Ekman Layer on Flows and Temperature of Viscoelastic Fluid in a Rotating Frame
  V. S. Rao Sasipalli, G. Shankar Sasipalli, G. Krishnam Raju et al.

Recent citations

- Analytical and Numerical Study on Cross Diffusion Effects on Magneto-Convection of a Chemically Reacting Fluid with Suction/Injection and Convective Boundary Condition
  Sheniyappan Eswaramoorthi *et al*

- Passive and Active Control on 3D Convective Flow of Viscoelastic Nanofluid With Heat Generation and Convective Heating
  S. Eswaramoorthi and M. Bhuvaneswari

- Thermal radiation and cross diffusion effects on 3D convective flow of a viscoelastic fluid over a stretchy paper with chemical reaction
  M Bhuvaneswari *et al*

Effect of partial slip and chemical reaction on convection of a viscoelastic fluid over a stretching surface with Cattaneo-Christov heat flux model

S Eswaramoorthi[1], M Bhuvaneswari[2], S Sivasankaran[2], H Niranjan[3] and S Rajan[4]

[1]Department of Mathematics, Dr.N.G.P. Arts and Science College, Coimbatore, Tamil Nadu, India

[2]Department of Mathematics, King Abdulaziz University, Jeddah, Saudi Arabia.

[3]Department of Mathematics, School of Advanced Sciences, VIT University, Vellore-632014, India

[4]Department of Mathematics, Erode Arts and Science College, Erode, Tamil Nadu, India

E-mail: niranjan.hari@vit.ac.in

Abstract. This article explores the effect of homogeneous-heterogeneous chemical reaction and partial slipon convective flow of a viscoelastic fluid with Cattaneo-Christov heat flux modelin the presence of suction/injection and convective boundary condition. The governing system of non-linear partial differential equations arereformed into ordinary differential equations with the help of similarity variables and then they are solved using homotopy analysis method. It is found that the surface heat transfer rate enhances on increasing the thermal relaxation time parameter and the surface mass transfer rate improved by increasing the slip parameter and homogeneous chemical reaction parameter.

1.Introduction

The study of viscoelastic fluid flow over a stretching surface has attracted many researchers because of its practical applications in engineering and industrial areas. Such applications are paper production, stretching of plastic film, glass fiber, crystal growing, wire drawing, hot rolling, etc. The heat transfer of a viscoelastic fluid over a stretching surface was studied by Cortell[1]. He found that the skin friction coefficient reduces with enhancing the viscoelastic parameter. Other related studies in this direction was found inRushi Kumar and Sivaraj [2], Eswaramoorthi*et al.* [3&4] and Sathish Kumar *et al.*[5].

The heat transfer is essentialprocess in many engineering and industrial applications. Fourier [6]was initially studied the heat transfer phenomenon in the form of parabolic energy equation for temperature field. One of the significant inadequate of this model is that the whole system is initially affected by the initial disturbance. So, Cattaneo[7] modified the Fourier's model including the relaxation time to avoid the paradox of heat conduction. Then, Christov [8] improved the Cattaneo model, namely, Cattaneo-Christov heat flux model to thermal relaxation time with Oldroyd's upper-convected derivatives to accomplish the material invariant formulation. Boundary layer flow of a

viscoelastic fluid with Cattaneo-Christov heat flux model was investigated by Hayat*et al.*[9]. Study of convective flow and heat transfer with chemical reaction is used in many chemical engineering/industrial processes,such as, fog formation, food processing, manufacturing of ceramics, drying, evaporation, groves of fruit trees and crops damage via freezing. Some of the studies in this directions are found in Karthikeyan*et al.* [10], Ruhaila*et al.* [11], Sivasankaran*et al.*[12].

The main purpose of this paper is to investigate the convective heat and mass transfer of a viscoelastic fluid flow over a stretching sheet with Cattaneo-Christov heat flux model. Effects of convective boundary condition,partialslip and suction/injection are examined.

2. Mathematical formulation

Let us presume the steady two-dimensional boundary layer flow of a viscoelastic fluid with Cattaneo-Christov heat flux model over a linear stretching sheet with partial slip and convective boundary condition. Assume that the sheet is moving with velocity $u = U_w = ax$, a is the positive constant. The sheet is kept at a constant temperature T_w which is higher than the ambient temperature of the fluid,T_∞. The effect of homogeneous and heterogeneous chemical reactions are considered in the fluid flow. The homogeneous chemical reaction for the cubic auto catalysis is $A + 2B \rightarrow 3B$ at rate $k_c ab^2$ and the simple first order heterogeneous chemical reaction at the surface is $A \rightarrow B$ at rate $k_s a$, where a and b are the constants of the chemical species A and B, k_c and k_s are the rate constants of the chemical species A and B. Also, we assume that both reaction processes are isothermal and there is no change in temperature profile. The bottom of the sheet is heated by convection from hot fluid with temperature T_f which yields the heat transfer coefficient h_c. The fluid experiences a first-order slip at the sheet surface. Under these assumptions, the governing boundary layer equations of this problem can be written in the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} + k_0\left(u\frac{\partial^3 u}{\partial x\partial y^2} + \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right) \tag{2}$$

$$\rho c_p V.\nabla T = -\nabla.q \tag{3}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_c ab^2 \tag{4}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_c ab^2 \tag{5}$$

where u and v are the velocity components in x and y directions, $v$is the kinematic viscosity, k_0 is the viscoelastic parameter, $c_p$is the specific heat, q is the heat flux, D_A and D_B are the respective diffusion species coefficients of A and B.The heat flux q satisfy the following equation

$$q + \lambda_1\left(\frac{\partial q}{\partial t} + V.\nabla q - q.\nabla V + (\nabla.V)q\right) = -k\nabla T \tag{6}$$

whereλ_1 and k are the relaxation time of heat flux and thermal conductivity of the fluid, respectively. For $\lambda_1 = 0$, the above equation reduces to classical Fourier's law. The fluid is incompressibility when form$\nabla.V = 0$and we have

$$q + \lambda_1\left(\frac{\partial q}{\partial t} + V.\nabla q - q.\nabla V\right) = -k\nabla T \tag{7}$$

Eliminating $q$between (3) and (7), we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \lambda_1\left(u^2\frac{\partial^2 T}{\partial x^2} + v^2\frac{\partial^2 T}{\partial y^2} + \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right]\frac{\partial T}{\partial x} + 2uv\frac{\partial^2 T}{\partial x\partial y} + \left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right]\frac{\partial T}{\partial y}\right) = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The boundary conditions of the above model are

$$u = U_w = ax + L\frac{\partial u}{\partial y}, v = V_w, -k\frac{\partial T}{\partial y} = h_c(T_f - T_\infty), D_A\frac{\partial a}{\partial y} = k_s a, D_B\frac{\partial b}{\partial y} = -k_s a \; at \; y = 0$$

$$u \to 0, \frac{\partial u}{\partial z} \to 0, T \to T_\infty, a \to a_0, b \to 0 \ as \ y \to \infty$$

Define the following similarity variables

$$\eta = \sqrt{\frac{a}{v}} y, u = ax f'(\eta), v = -\sqrt{av} f(\eta), \theta = \frac{T - T_\infty}{T_f - T_\infty}, a = a_0 \phi, b = a_0 h \tag{9}$$

Substituting Equation (9) intotheEquations (2), (4), (5) and (8), we have

$$f''' + ff'' - f'^2 + K\left(2f'f''' - f''^2 - ff^{iv}\right) = 0 \tag{10}$$

$$\frac{1}{Pr}\theta'' + f\theta' - \gamma(ff'\theta' + f^2\theta'') = 0 \tag{11}$$

$$\frac{1}{Sc}\phi'' + f\phi' - k_1\phi h^2 = 0 \tag{12}$$

$$\frac{\delta}{Sc}h'' + fh' + k_1\phi h^2 = 0 \tag{13}$$

The boundary conditions become,
$f(0) = f_w, f'(0) = 1 + \lambda f''(0), f'(\infty) = 0, f''(\infty) = 0, \quad \theta'(0) = -Bi\,\theta(0),$
$\theta(\infty) = 0, \phi'(0) = k_2\phi(0), \delta h'(0) = -k_2\phi(0), \phi(\infty) = 0, h(\infty) = 0,$ (14)
where$K = \frac{k_0 a}{v}$ is the viscoelastic parameter, $\lambda = L\sqrt{\frac{a}{v}}$ is the slip parameter, $Pr = \frac{v\rho c_p}{k}$ is the Prandtlnumber, $\gamma = \lambda_1 a$ is the thermal relaxation time parameter, $Sc = \frac{v}{D_A}$ is the Schmidt number, $k_1 = \frac{k_c a_0^2}{a}$is the strength of homogeneous chemical reaction, $fw = \frac{V_w}{\sqrt{av}}$ is the suction/injection parameter, $Bi = \frac{h_c\sqrt{\frac{v}{a}}}{k}$ is the Biot number, $k_2 = \frac{k_s\sqrt{v}}{D_A\sqrt{a}}$ is the strength of heterogeneous chemical reaction and $\delta = \frac{D_B}{D_A}$ is the ratio of diffusion coefficient. The diffusion coefficients are comparable size and we assume that these coefficient are equal, i.e. $\delta = 1$. Thus
$$\phi(\eta) + h(\eta) = 1$$
Then the equation becomes
$$\frac{1}{Sc}\phi'' + f\phi' - k_1\phi(1 - \phi)^2 = 0 \tag{15}$$
with the boundary conditions$\phi'(0) = k_2\phi(0)$and$\phi(\infty) \to 1$.

The skin friction coefficient, local Nusselt number and local Sherwood number are important physical parameters.The reduced skin friction coefficient, local Nusselt number and local Sherwood number are given by

$$\frac{1}{2}C_f\sqrt{Re} = (1 + 3K)f''(0), Nu/\sqrt{Re} = -\theta'(0) \text{and} Sh/\sqrt{Re} = -\phi'(0).$$

3. HAM solutions
The initial approximations for homotopy analysis solutions are chosen as $f_0 = f_w + \frac{1 - e^{-\eta}}{1 + \lambda}; \theta_0 = \frac{Bie^{-\eta}}{1 + Bi}; \phi_0 = 1 - \frac{1}{2}e^{-k_2\eta}$, the auxiliary linear operators L_f, L_θ and L_ϕ are defined as $L_f = f''' - f'; L_\theta = \theta'' - \theta; L_\phi = \phi'' - \phi$with satisfying the following properties

$$L_f[C_1 + C_2 e^\eta + C_3 e^{-\eta}] = 0; L_\theta[C_4 e^\eta + C_5 e^{-\eta}] = 0; L_\phi[C_6 e^\eta + C_7 e^{-\eta}] = 0, \quad \text{where} C_i, (i = 1 - 7)$$
denote the arbitrary constants.

The general solution of the Equations (10)-(11) and (15) are

$$f_m(\eta) = f^*{}_m(\eta) + C_1 + C_2 e^{\eta} + C_3 e^{-\eta}, \theta_m(\eta) = \theta^*{}_m(\eta) + C_4 e^{\eta} + C_5 e^{-\eta}$$
$$\phi_m(\eta) = \phi^*{}_m(\eta) + C_6 e^{\eta} + C_7 e^{-\eta}$$

where$f^*{}_m(\eta)$, $\theta^*{}_m(\eta)$ and $\phi_m(\eta)$ are the special solutions.These general solutions contain the auxiliary parameters h_f, h_θ and h_ϕ. These parameters are adjusting and controlling the convergence of the desired solutions. The h_f, h_θ andh_ϕ curves are plotted in Figure 1. It is observed fromtherange values of h_f, h_θ and h_ϕ ($-1.8 \le h_f \le -0.3$ & $-2 \le h_\theta, h_\phi \le -0.1$) that the HAM solution convergence in the region of η when $h_f = h_\theta = h_\phi = -1$.

Table 1 presents the comparison of skin friction coefficient between our results and published results. It is seen that from the table that our results are in good agreement with previous results.

4. Correlation
The correlation equations of the skin friction, Nusselt number and Sherwood number are derivedas

$$\frac{1}{2} C_f \sqrt{Re} = -0.580474 - 1.210071K - 0.175144 fw + 0.156614\lambda - 0.000515Bi - 0.000527\gamma$$
$$- 0.000527 k_1 - 0.003905 k_2$$
$$Nu/\sqrt{Re} = 0.236642 + 0.019823K + 0.181523 fw - 0.017403\lambda + 0.036940Bi + 0.018367\gamma$$
$$- 0.001836 k_1 - 0.013605 k_2$$
$$Sh/\sqrt{Re} = -0.177937 - 0.035976K - 0.247830 fw + 0.021479\,\lambda - 0.000670Bi - 0.000686\,\gamma$$
$$+ 0.097694 k_1 - 0.116217 k_2$$

5.Results and Discussion
In this section, we present the velocity, temperature, concentration, skin friction coefficient, local Nusselt number and local Sherwood number for different values of pertinent parameters with the fixed values of the Prandtl number ($Pr = 0.9$) and Schmidt number ($Sc = 0.9$).Figures 2(a and b) illustrate the influence of slip parameter (λ) on velocity profile for Newtonian and viscoelastic fluids with suction/injection. It is found that the fluid velocity and its momentum boundary layer thickness suppress on increasing the values of slip parameter. Variations of the slip parameter on temperature profile for Newtonian and viscoelastic fluids are shown in Figures 3(a andb). It observed that the thermal boundary layer thickness enlarges with increasing the slip parameter. Figures 4(a andb) show the effect of the Biot number and thermal relaxation time parameter on temperature profile. It is concluded that the thermal boundary layer thickness boost up on increasing the Biot number and it supresses with increasing the thermal relaxation time parameter.

Concentration profile for different values of slip parameter for Newtonian and viscoelastic fluids are displayed in Figures 5(a andb). It is found that the concentartion suppresses by rising the values of the slip parmeter. In addition, the boundary layer thickness of viscoelastic fluid is higher than that of the Newtonian fluid.Figure 6(a) depict the skin friction coefficient for various values on K, fw and λ. It is found that the skin friction reduces with increasing the values of viscoelatic parameter and suction/injection parameter.It enhances on increasing the slip parameter. The local Nusselt number for different values of thermal relaxation time parameter, suction/injection parameter and slip parameter are shown in Figure 6(b). It is found that the surface heat transfer rate diminishes on growing the slip parameter and suction/injection parameter. However, it is an increasing function of thermal relaxation time parameter. Figures 7(a andb) explore the local Sherwood number for different values of slip, suction/injection, homogeneous and heterogeneous chemical reaction parameters. It is seen that the surface mass transfer rate drops with increasing the suction/injection parameter and heterogeneous chemical reaction parameter.Itaugments on increasing the slip parameter and homogeneous chemical reaction parameter.

6. Conclusion
In this article, the effects of homogeneous and heterogeneous chemical reactions and partial slip on convective flow of viscoelastic fluid with Cattaneo-Christov heat flux model in the presence of convective boundary conditions and suction/injection are explored. The velocity reduces by increasing

the values of slip parameter, suction/injection parameter.The thermal boundary layer thickness enlarges with increasing the slip parameter, Biot number and it reduces on increasing the thermal relaxation time parameter and viscoelastic parameter.Thesolutal boundary layer thickness improves with viscoelastic parameter and suction/injection parameter. The skin friction coefficient falls with increasing the viscoelastic parameter and suction/injection parameter.The heat transfer rate enhances on increasing the thermal relaxation time parameter.The mass transfer rate improves by increasing the slip parameter and homogeneous chemical reaction parameter and it reduces by increasing the suction/injection parameter and heterogeneous chemical reaction parameter.

Table 1. Computations showing the comparison with Hayat*et al.* [9] for different values of K with $fw = 0.2$ and $\lambda = 1.0$.

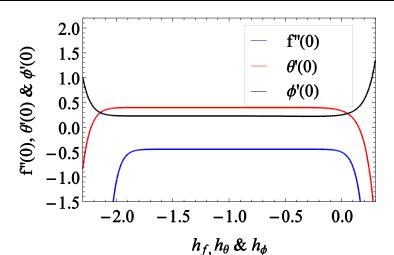| K	-0.3	-0.2	-0.1	0	0.1	0.2	0.3
Present study	-0.11952	-0.44721	-0.73787	-1	-1.23950	-1.46059	-1.66641
Hayat *et al.* [9]	-0.11952	-0.44721	-0.73786	-1	-1.23950	-1.46059	-1.66641

Figure 1.$h$curves of $f''(0), \theta'(0)$ and $\phi'(0)$with $K = 0.2, fw = 0.2, \lambda = 1.0, \gamma = 0.5$, $Bi = 0.5 k_1 = 0.5$ and $k_2 = 0.5$.
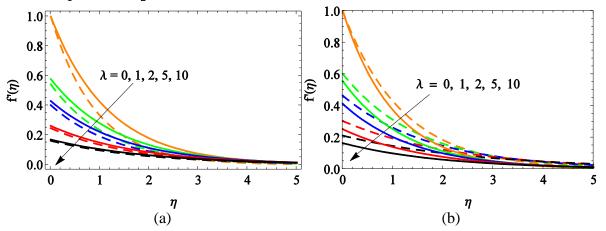

Figure 2. Velocity profile for different values of λ with (a) $K = 0$(dashed line), $K = 0.2$(solid line) and $fw = 0.2$ and (b) $fw = -0.2$(dashed line), $fw = 0.2$(solid line) and $K = 0.2$.
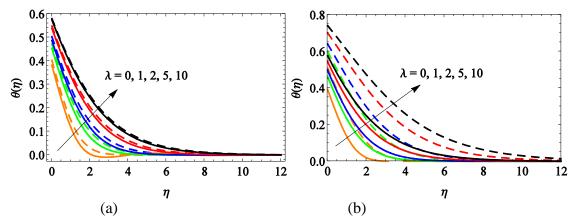
Figure 3. Temperature profile for different values of λ with (a)$K = 0.0$(dashed line), $K = 0.2$(solid line), $fw = 0.2$, $\gamma = 0.5$ and Bi=0.5 and (b) $fw = -0.2$(dashed line), $fw = 0.2$(solid line), $K = 0.2$,$\gamma = 0.5$ and $Bi = 0.5$.
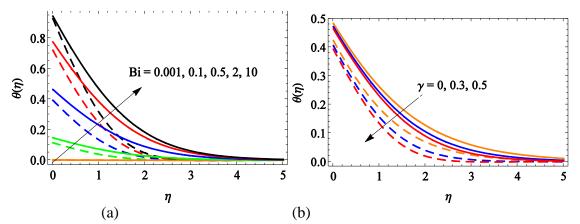

Figure 4. Temperature profile for different values (a)Bi with $\lambda = 0.0$(dashed line), $\lambda = 1.0$(solid line), $fw = 0.2$, $K = 0.2$ and$\gamma = 0.5$and (b) γ with $\lambda = 0.0$(dashed line), $\lambda = 1.0$(solid line), $fw = 0.2$, $K = 0.2$and$Bi = 0.5$.
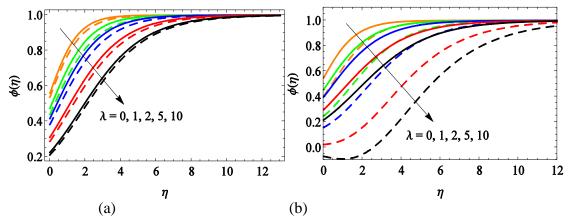

Figure 5. Concentration profile for different values of $\lambda$with (a)$K = 0$(dashed line), $K = 0.2$(solid line)with $fw = 0.2, k_1{=}0.5$ and $k_2 = 0.5$ and (b) $fw = -0.2$(dashed line), $fw = 0.2$(solid line) $K = 0.2$, $k_1{=}0.5$ and $k_2 = 0.5$

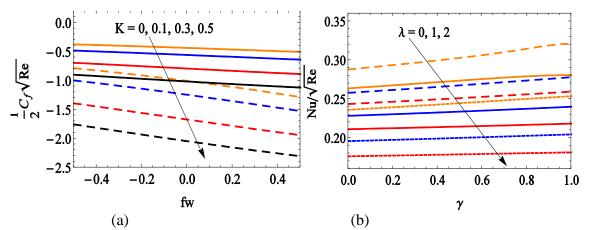(a)   (b)

Figure 6. Skin friction coefficient for $\lambda = 0.0$(dashed line), $\lambda = 1.0$(solid line) with various values of K and fw (b) local Nusselt number for $fw = -0.2$(dashed line), $fw = 0.0$(solid line) $fw = 0.2$(dotted line) for various values of λ and γ and $Bi = 0.5$.

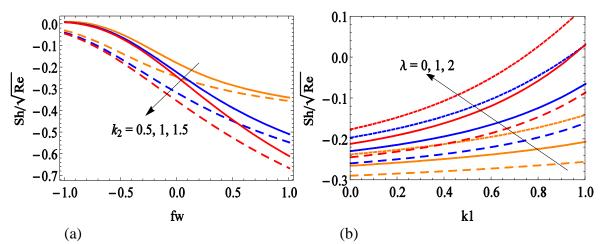(a)   (b)

Figure 6. Local Sherwood number (a) $\lambda = 0.0$(dashed line), $\lambda = 1.0$(solid line) with various values of fw and k_2 for $K = 0.2$ and $k_1 = 0.5$ (b) $fw = -0.2$(dashed line), $fw = 0.0$(solid line) $fw = 0.2$(dotted line) for various values of k_1, λ for $K = 0.2$ and $k_2 = 0.5$.

References

[1]   CortellR2006*Int. J.Nonlin. Mech.* **41** 78–85
[2]   Rushi Kumar B and Sivaraj R 2013*Int. Communi. Heat Mass Transf.***40**1-6
[3]   Eswaramoorthi S, Bhuvaneswari M, SivasankaranS and Rajan S2015 *Procedia Engi.***127**916-923.
[4]   Eswaramoorthi S, Bhuvaneswari M, SivasankaranS and Rajan S 2016*Scientia Iranica B***23**2575-86.
[5]   Sathish Kumar M, Sandeep N, Rushi Kumar B and Dinesh PA 2017*Alexandria Engin. J.In Press*
[6]   Fourier JBJ 1948TheorieAnalytique De La Chaleur Chez FirminDidot Paris 1822
[7]   Cattaneo C 1948 Sulla conduzionedelcaloreAtti. Semin. Mat. Fis Univ. Modena Reggio Emilia**3**83-101
[8]   Christov CI 2009 *Mech. Res. Commun.***36**481-486
[9]   Hayat T, Muhammad T, Alsaedi A and MustafaM 2016*PLoS ONE* **11** 1-11

[10]   Karthikeyan S,Bhuvaneswari M, Sivasankaran S and Rajan S 2016*J.Appl.Fluid Mech.***9** 1447-1455

[11]   Ruhaila M K, Sivasankaran S, Bhuvaneswari M and SiriZ 2016*J. Appl. Fluid Mech.* **9** 379-388

[12]   Sivasankaran S, Niranjan H andBhuvaneswari M 2017*Int. J. Numer.Methods Heat Fluid Flow***27**454-470