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Finite Element Vibration Analysis of a Magnetorheological Fluid Sandwich Beam

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Abstract

This study investigates the dynamic properties of a sandwich beam with magnetorheological (MR) fluid as a core material between the two layers of the continuous elastic structure. Timoshenko beam theory has been employed to derive the governing differential equation of motion in the finite element form for the transverse vibration response of MR fluid sandwich beam. The validation of the developed finite element formulation is demonstrated by comparing the results in terms of natural frequencies evaluated with those of available literature. Various parametric studies are also performed in terms of variations of the natural frequencies and loss factor as functions of the applied magnetic field and thickness of the MR fluid layer for various boundary conditions.

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Keywords: MR fluid; Sandwich beam; Magnetic field; Natural frequency; Loss factor.

1. Introduction

Vibration control of continuous elastic structures has been an active subject of research for the last few decades. The active vibration control concepts have evolved to achieve enhanced vibration suppression of structures and adapt to changes in the excitation and structural properties [1-3]. Compared with passive systems, active vibration control system improves the performance in terms of reduction in vibration amplitude of the structures. They, however, could not be justified for application when the added cost and power requirements prevail the performance gains. Alternatively, a range of semi-active vibration control concepts have evolved for various structural vibration control applications, which could offer better performance compared to both active and passive

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systems and is economical, safe and does not require either high power actuators or large power supply [4-6]. Fluids with controllable rheology and thus the damping properties, such as electrorheological (ER) and magnetorheological (MR) fluids, have been used in various semi-active vibration control applications [7, 8]. Such fluids exhibit rapid change in their rheological, damping and stiffness properties with application of an electric or magnetic field, respectively [9]. However, the ER fluids exhibit a number of shortcomings compared to the MR fluids including low yield strength, requirement of high voltage and greater sensitivity to common impurities. On the other hand, the MR fluids are known to exhibit considerably higher dynamic yield strength and greater insensitivity to temperature variation and contaminants compared to ER fluids [9, 10]. The yield stress of the MR fluid lies in the order of 2-3 kPa in the absence of a magnetic field and it rapidly exceeds 80 kPa under the application of a magnetic field in the order of 3000 Oe [8]. These fluids are also considered to yield high bandwidth control through rapid variations in the rheological properties under a varying magnetic field. The MR fluids have thus been considered promising for various semi-active vibration control applications in the low to moderate frequency ranges, including automotive suspension, structures, etc [11-13].

In the past few years, the vast majority of the efforts in MR fluids have been focused on designs of MR and ER dampers and evaluations of their potential benefits in vibration suppression in structures and systems [13-15]. The ER/MR fluid damping has been employed in various structural applications either through lumped damping elements at selected discrete locations [14] or through continuous MR/ER fluid layer treatments in selected structural members [16, 17]. The former approach may require multiple damping elements since suppression in structures may involve consideration of multiple modes of vibration [15]. This may lead to higher weight. Alternatively, the ER/MR sandwich structures achieved by embedding ER/MR material layers between two elastic metal layers can yield distributed control force along the structural member treated. This approach can ease control of structure vibration over a broad range of frequencies through variations in distributed stiffness and damping properties in response to applied electric or magnetic field.

The dynamic properties of ER fluid based sandwich structures have been analyzed in large number of studies [18, 19], however very few studies have focused on investigating the dynamic characterization of MR fluid based sandwich structures. The dynamic responses of a MR fluid adaptive structure was investigated by Yalcintas and Dai [20] using the energy approach and compared the responses with those of a structure embedded with ER fluid. It was concluded that the natural frequencies of MR fluid based adaptive structure could be nearly twice those of the ER fluid based adaptive structure. The analytical solution for the dynamic responses of a MR fluid sandwich beam was developed by Sun et al. [21] using an energy approach and compared the results in terms of natural frequencies with those of experimental results. Yeh and Shih [22] analyzed the dynamic characteristics and instability of MR adaptive structures under buckling loads based on the DiTaranto [23] sixth-order partial differential equation together with the incremental harmonic balance method. Rajamohan et al. [24, 25] derived finite-element and Ritz formulations for a sandwich beam with uniform and partial MR-fluid treatment and demonstrated their validity through experiments conducted on a cantilever sandwich beam. It was demonstrated that the natural frequencies increase with increase in magnetic field. Prieto et al. [26] experimentally investigated the dynamic responses of a MR-sandwich cantilever beam subject to a uniform and non-uniform magnetic field. It was concluded that the natural frequency of the beam decreases as the permanent magnets are moved away from the fixed support. The influence of locations of the MR fluid segments on the modal damping factor was further investigated under different end conditions using modal strain energy approach and finite element method by Rajamohan et al. [27]. Optimal configurations of a partially treated MR sandwich beam were subsequently identified to achieve maximum modal damping factor corresponding to the first five flexural modes, considered either individually or simultaneously. Rajamohan et al. [28] also investigated the full state and limited state flexible mode shape (FMS) based controllers for the suppression of transient and forced vibration of a cantilever beam with full and partial magnetorheological (MR) fluid treatments. Rajamohan and Ramamoorthy [29] studied the vibration behavior of a non-homogeneous MR fluids based sandwich beam. It is to be noted that all the above studies have considered the elastic layers as Euler's beam and the developed formulations are applicable for thin elastic structures. The dynamic characteristics of the sandwich beam with thick layers of elastic layers embedded with MR fluid have not yet been explored.

In this present study, the governing equations of a sandwich beam structure employing an MR fluid as the core layer are presented in the finite element form. Timoshenko beam theory has been employed to derive the governing differential equation of motion in the finite element form for the transverse vibration response of MR fluid sandwich beam. The validation of the developed finite element formulation is demonstrated by comparing the results in terms of natural frequencies evaluated with those of available literature. Various parametric studies are also performed in terms of variations of the natural frequencies and loss factor as functions of the applied magnetic field and thickness of the MR fluid layer for various boundary conditions.

2. Mathematical Modelling of the MR Fluid Sandwich Beam

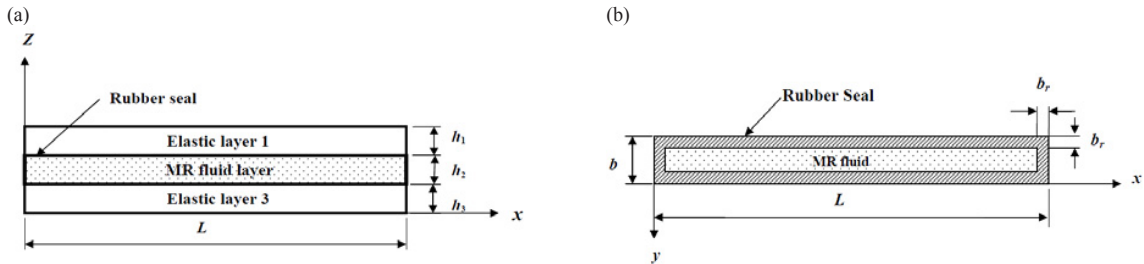


Fig. 1. (a). The MR fluid sandwich beam; (b) Plane view of the MR fluid layer

A sandwich beam with MR fluid as the core material and elastic layers as the face material (Fig. 1(a)) is considered for development of the finite element model. As the Young’s modulus of the MR fluid is nearly negligible compared to that of the elastic layers, the normal stresses in the fluid layer are neglected. It is also considered that the flexural wavelengths of the face layers are less than ten times the cross sectional dimensions and thus the shear deformation and rotary inertia effects of the face layers are included in the formulation. Hence, the plane sections which are normal to the deformed centroidal axis does not remain plane after bending. The damping due to elastic layers is also assumed to be negligible. Furthermore, the transverse displacement w in a given cross-section is assumed to be uniform. Let the longitudinal displacements of the mid-planes of the elastic layers in the x -direction be u_1 and u_3 and the longitudinal displacement component of any point in the MR fluid be u . The mid-layer is further assumed as a neutral layer in the transverse plane. The top and bottom surfaces are thus considered to undergo axial compression and tension, respectively. Consequently, the axial displacement of the sandwich beam is considered to be equivalent to the axial displacement at the MR fluid layer of the beam. The shear strain γ in the MR layer can be derived from [30]:

$$\gamma = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \tag{1}$$

where

$$\frac{\partial u}{\partial z} = \frac{(h_1 + h_3)}{2h_2} \frac{\partial w}{\partial x} + \frac{(u_1 - u_3)}{h_2} \tag{2}$$

which yields the shear strain as a function of the layer thickness as:

$$\gamma = \frac{D}{h_2} \frac{\partial w}{\partial x} + \frac{u_1 - u_3}{h_2} \tag{3}$$

where $D = h_2 + \frac{1}{2}(h_1 + h_3)$.

Let the longitudinal forces in each of the elastic layers be denoted by F_1 and F_3 with their lines of action in the midplanes of the elastic layers. These forces can be expressed as

$$F_1 = E_1 A_1 \frac{\partial u_1}{\partial x}; \quad F_3 = E_3 A_3 \frac{\partial u_3}{\partial x} \quad (4)$$

where A_1 and A_3 are the cross-section areas of layers 1 and 3, respectively and E_1 and E_3 are the corresponding Young's moduli. Since the beam is assumed to be free of longitudinal forces, i.e., $F_1 + F_3 = 0$, Eq. (4), yields the following relationship between the longitudinal deflection of the elastic layers:

$$E_1 A_1 \frac{\partial u_1}{\partial x} = -E_3 A_3 \frac{\partial u_3}{\partial x}. \quad (5)$$

By integrating with respect to x , the above relation can be simply expressed as a function of the longitudinal deflections:

$$u_3 = -e u_1 \quad (6)$$

where $e = \frac{E_1 A_1}{E_3 A_3}$.

A sealant material, Buna-N rubber, is also considered around the edges of the MR fluid layer to ensure uniform layer thickness and containment of the MR fluid within the sandwich beam (Fig. 1(b)). The mid-layer of the sandwich beam comprising the rubber seal and the MR fluid, however, is modelled as a homogeneous material layer with equivalent shear modulus expressed by moduli and widths of the two materials, such that:

$$\bar{G} = G_r \left(\frac{b_r}{b} \right) + G^* \left(1 - \frac{b_r}{b} \right) \quad (7)$$

where \bar{G} is the equivalent shear modulus of the homogeneous layer, b_r and b are the widths of the rubber and entire beam, respectively, G_r and G^* are the shear modulus of the rubber and MR fluid, respectively. In the pre-yield regime, the MR material demonstrates viscoelastic behavior, which has been described in terms of the complex modulus G^* and given by [31],

$$G^* = G' + iG'' \quad (8)$$

where G' is storage modulus of the MR fluid, which is related to the average energy stored per unit volume of the material during a deformation cycle, and G'' is the loss modulus, a measure of the energy dissipated per unit volume of the material over a cycle.

2.1. The energy equations

The governing equations of motion for the MR sandwich beam are formulated in the finite element form using Lagrange's energy approach. To accomplish this, the total strain and kinetic energy of the system are derived. The strain energy due to elastic layers, $V_{1,3}$, can be derived as the summation of the elastic energy due to axial, transverse and shear deformations and are presented as:

$$V_{1,3} = \frac{1}{2} \int_0^L (E_1 A_1 + E_3 A_3 e^2) \left(\frac{\partial u}{\partial x} \right)^2 dx + \frac{1}{2} \int_0^L (E_1 I_1 + E_3 I_3) \left(\frac{\partial \theta}{\partial x} \right)^2 dx + \frac{1}{2} \int_0^L (\kappa_1 A_1 G_1 + \kappa_3 A_3 G_3) \left(\frac{\partial w}{\partial x} - \theta \right)^2 dx \quad (10)$$

where I_1 and I_3 are the second moments of inertia at the centroids of elastic layers 1 and 3, respectively and G_1 , G_2

and κ_1 and κ_3 are the shear modulus and shear correction factors of the elastic layers 1 and 3, respectively. The shear strain energy of the mid-layer comprising the MR fluid and the rubber compound is obtained as [24]

$$V_2 = \frac{1}{2} \int_0^L \bar{G} A_2 \left[\frac{D}{h_2} \frac{\partial w}{\partial x} - \frac{(1+e)u}{h_2} \right]^2 dx \tag{11}$$

where $A_2 = b \times h_2$ is the total cross sectional area of layer 2 which includes both rubber material and MR fluid. The total strain energy V of the sandwich beam structure is the sum of those due to elastic and fluid layers, such that

$$V = V_1 + V_2 + V_3 \tag{12}$$

The kinetic energy includes those associated with: (i) the transverse motion due to translation and rotation of the elastic layers and the MR layers (T_1); (ii) the axial deformations of the elastic layers (T_2); and (iii) the rotational deformation of the MR layer due to the strain displacement (T_3). The kinetic energy associated with the transverse motions of the elastic and fluid layers, T_1 , and axial deflections of the elastic layers, T_2 , can be derived as:

$$T_1 = \frac{1}{2} \int_0^L (\rho_1 A_1 + \rho_2 A_2 + \rho_r A_r + \rho_3 A_3) \left(\frac{\partial w}{\partial t} \right)^2 dx + \frac{1}{2} \int_0^L (\rho_1 I_1 + \rho_3 I_3) \left(\frac{\partial \theta}{\partial t} \right)^2 dx \tag{13}$$

$$T_2 = \frac{1}{2} \int_0^L (\rho_1 A_1 + e^2 \rho_3 A_3) \left(\frac{\partial u}{\partial t} \right)^2 dx \tag{14}$$

where ρ_1 and ρ_3 are the mass densities of the elastic layers, ρ_2 and ρ_r are the mass densities of the fluid and the rubber materials, respectively. The kinetic energy associated with the rotation of the MR fluid layer, T_3 , can be expressed as [24]

$$T_3 = \frac{1}{2} \int_0^L \rho_2 I_2 \left[\frac{-(1+e)}{h_2} \left(\frac{\partial u}{\partial t} \right) + \frac{D}{h_2} \left(\frac{\partial^2 w}{\partial x \partial t} \right) \right]^2 dx \tag{15}$$

where I_2 is the second moment of inertia at the centroid of the MR fluid layer. The total kinetic energy T of the sandwich beam can then be obtained from

$$T = T_1 + T_2 + T_3 \tag{16}$$

Apart from the strain and kinetic energies, the work done by the excitation force, if present, also needs to be considered in the formulation.

2.2. Finite element formulations

In the finite element formulation, a thick beam element with two end nodes and three DOF per node is considered. The DOF include the axial u , transverse w and rotational θ displacements of the beam. The axial, transverse and rotational displacements can be expressed in terms of nodal displacement vectors and shape functions, as

$$u(x,t) = N_u(x) \{d(t)\} \tag{17}$$

$$w(x,t) = N_w(x) \{d(t)\} \tag{18}$$

$$\theta(x,t) = N_\theta(x) \{d(t)\} \tag{19}$$

where displacement vector, $d(t) = \{u_1, w_1, \theta_1, u_2, w_2, \theta_2\}$ and $N_u(x)$ and $N_w(x)$ are the common linear and cubic polynomial beam shape functions.

Lagrange's equations are used to develop the governing differential equations in finite element form. In general form, Lagrange's equation can be written as

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i, \quad i=1, \dots, n \quad (20)$$

where n is the total DOF considered in the formulation and Q_i is the generalized force corresponding to the i^{th} DOF. Upon substituting for the energy expressions in Eq. (12) and Eq. (16) together with the deflection functions in Eq. (17), Eq. (18), and Eq. (19), the governing equations of motion for the MR sandwich beam element in the finite element form can be obtained as

$$[m]\{\ddot{d}\} + [k]\{d\} = \{f\} \quad (21)$$

where $[m]$ and $[k]$ are the element mass matrix and stiffness matrix, respectively and $\{f\}$ is the element force vector. Assembling the mass and stiffness matrices and the force vector for all the elements yields the system of governing equations of motion of the MR sandwich beam in the finite element form, which can be expressed in the following general form:

$$[M]\{\ddot{d}\} + [K]\{d\} = \{F\} \quad (22)$$

where $[M]$, $[K]$ and $\{F\}$ are the system mass and stiffness matrices, and the force vector, respectively.

3. Validation

The effectiveness of the developed finite element formulation for a MR fluid sandwich beam is demonstrated by comparing the results in terms of natural frequencies available in literature [24]. In the literature [24], the finite element formulation has been developed for a MR fluid sandwich beam neglecting the shear deformation and rotary inertia effects of the elastic face layers to facilitate the investigation of the dynamic properties of a thin face layers based MR fluid sandwich beam. Even though the finite element formulation in the present study accommodates the shear deformation and rotary inertia effects existing in the face layers, the validation of the developed finite element formulation is demonstrated by comparing the natural frequencies evaluated in literature [24]. The simulation is performed for a thin face layers based MR fluid sandwich beam with dimensions of elastic layers $300 \times 30 \times 1$ mm with identical thickness of MR fluid layer. The various material properties considered are: $E_1 = E_3 = 68$ GPa; $G_1 = G_3 = 26$ GPa; $\rho_1 = \rho_3 = 2700$ kg/m³; $\rho_2 = 3500$ kg/m³; $\rho_r = 1233$ kg/m³. The shear modulus functions of MR fluid considered for the simulation is [24]:

$$\begin{aligned} G'(B) &= -3.3691B^2 + 4.9975 \times 10^3 B + 0.893 \times 10^6 \\ G''(B) &= -0.9B^2 + 0.8124 \times 10^3 B + 0.1855 \times 10^6 \end{aligned} \quad (23)$$

where B is the applied magnetic field intensity in gauss (G). The first five natural frequencies are evaluated at two different magnetic fields under simply supported at both ends and the results are presented together with the percent deviation in Table 1. It can be seen that a good agreement has been observed between the results obtained in the literature [24] and present finite element formulation in which the shear deformation and rotary inertia effects are included.

Table 1. Comparison of natural frequencies of the MR fluid sandwich beam obtained using the developed finite element method and with literature [24] under simply supported end conditions

Mode No.	Natural frequency (Hz)					
	Magnetic field = 0 G			Magnetic field = 250 G		
	Present FEM	Literature [24]	% deviation	Present FEM	Literature [24]	% deviation
1	40.74	40.31	1.06	51.88	50.92	1.85
2	105.70	103.10	2.46	123.56	120.32	2.62
3	206.51	200.07	3.12	229.01	222.24	2.96
4	344.72	332.45	3.56	369.67	357.33	3.34
5	521.57	501.67	3.82	547.94	528.15	3.61

4. Results and Discussion

The various parameters such as field intensity, fluid layer thickness, beam geometry, boundary conditions and shear corrections factors influence the dynamic properties of a MR fluid sandwich beam. The proposed finite element model which takes into account the shear deformation and the rotary inertia effects of the thick elastic layer is used to study the effects of variations in the magnetic field intensity and the MR fluid layer thickness on the properties of the beam in terms of natural frequencies and the loss factors for different boundary conditions. The simulation results are obtained by considering identical baseline thickness of 5 mm of the elastic and fluid layers, while the material and all other geometric properties of the layers are identical to those described previously in section 3.

4.1. Effect of the magnetic field intensity on natural frequencies and loss factors

The effect of magnetic field intensity on the variation of natural frequencies of thick face layers and MR fluid core layer sandwich beam is investigated by performing the simulation at various magnetic field intensities under various boundary conditions including simply supported (SSB), clamped free (CFB) and clamped-clamped (CCB) conditions. The results are presented in Table 2.

Table 2. Influence of variations in the magnetic field intensity on the natural frequencies of the MR fluid sandwich beam for different boundary conditions

Magnetic field intensity (G)	Boundary conditions	Natural frequency (Hz)				
		Mode				
		1	2	3	4	5
0	SSB	104.28	396.96	882.36	1557.60	2419.30
	CFB	42.38	227.44	616.92	1196.10	1962.90
	CCB	224.91	613.11	1193.10	1959.80	2909.90
100	SSB	106.72	399.54	884.98	1560.20	2421.90
	CFB	44.78	230.95	620.15	1199.10	1965.90
	CCB	226.33	615.05	1195.20	1962.10	2912.30
200	SSB	108.76	401.73	887.21	1562.50	2424.10
	CFB	46.65	233.87	622.88	1201.80	1968.50
	CCB	227.53	616.71	1197.10	1964.00	2914.30
300	SSB	110.43	403.54	889.05	1564.30	2426.00
	CFB	47.42	235.24	624.13	1203.90	1970.60
	CCB	228.52	618.08	1198.60	1965.60	2915.90

It can be realized from the results that the natural frequencies corresponding to all the modes increase with increase in the magnetic field, irrespective of the boundary conditions. A similar trend in the natural frequencies has also been reported in literature [24]. The increase in natural frequencies with increasing magnetic field is attributed to increase in the complex shear modulus of the MR fluid and thus the structure stiffness under higher magnetic field. It is also shown that the clamped-clamped condition yields the highest natural frequencies, while the natural frequencies of the clamped free beam are the lowest for all modes considered, irrespective of the magnetic field intensity. The results also suggest that an increase in the field intensity generally yields the largest stiffness increase for all the modes for the relatively soft clamped free condition, and relatively smallest change in stiffness for the relatively stiff clamped-clamped beam.

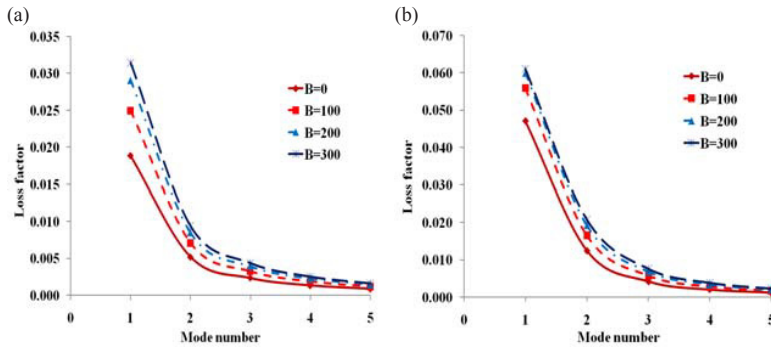


Fig.2. Influence of variations in the magnetic field intensity, B, on the loss factor corresponding to various modes under (a) Simply Supported Beam; (b) Clamped Free Beam

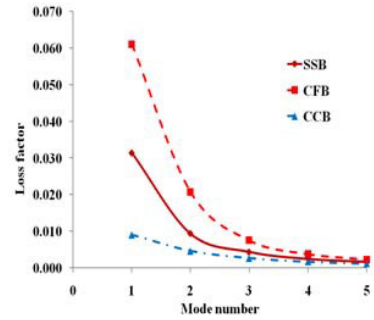


Fig. 3. Effect of boundary conditions on loss factors corresponding to the first five modes

The loss factor due to the MR fluid sandwich beam is computed as the ratio of the square of the imaginary component of the complex natural frequency to that of the real component [17, 26]. The variations in the loss factors of the beam with simply supported and clamped free boundary conditions for different field intensities are computed corresponding to the first five modes and are presented in Fig. 2. The results generally show an increase in the loss factors with increase in the magnetic field for both the boundary conditions. The loss factor is merely the ratio of energy dissipated per radian to the total strain energy V , both of which increase with the magnetic field. Furthermore, the dissipated energy is directly related to the loss modulus, which increases with the field intensity as seen in Eq. (23). The relative increase in the loss modulus and thus the dissipated energy with increase in the magnetic field, however, is greater than that in the total strain energy which leads to higher loss moduli under increasing magnetic field. It can also be observed that the loss factors decreases in higher modes compared to those of the fundamental mode. However, the similar trend could not be observed in clamped free end condition, particularly in fundamental mode, in literature [24] in which the shear deformation and rotary inertia effects of the elastic layers are neglected. Hence, it can be concluded that the loss factors could be increased with increase in magnetic field at all the modes considered irrespective of the boundary conditions if the elastic layers are thick with inclusion of the shear deformation and rotary inertia effects.

The influence of loss factors under various boundary conditions of MR fluid sandwich beam are investigated by performing the simulation under various boundary conditions at a magnetic field of 300 G and the results are shown in Fig. 3. It can be seen that the loss factors decrease at higher modes at all the boundary conditions considered. It can also be shown that clamped free end conditions yields highest loss factors and clamped-clamped beam yields the lowest loss factors at all the modes. It can be attributed to the fact that the strain energy of the clamped free beam is lower than those of simply supported and clamped-clamped beams.

4.2. Influences of the MR layer thickness on natural frequencies and loss factor

The effect of MR fluid layer thickness on the natural frequencies and the loss factor is investigated by evaluating the natural frequencies and loss factors at a magnetic field of 300 G under the clamped-clamped end condition at various thickness ratio of (h_2/h_1) and the results are shown in Fig. 4. The results generally show a decrease in the natural frequencies with increase in thickness of the MR layer. A significant variation can be observed in higher modes compared to that of lower modes. This can be attributed to the fact that the relative variation in the mass of the structure is higher than that of the stiffness when the thickness of the MR fluid layer increases. Such variation has also been reported in the literature [24].

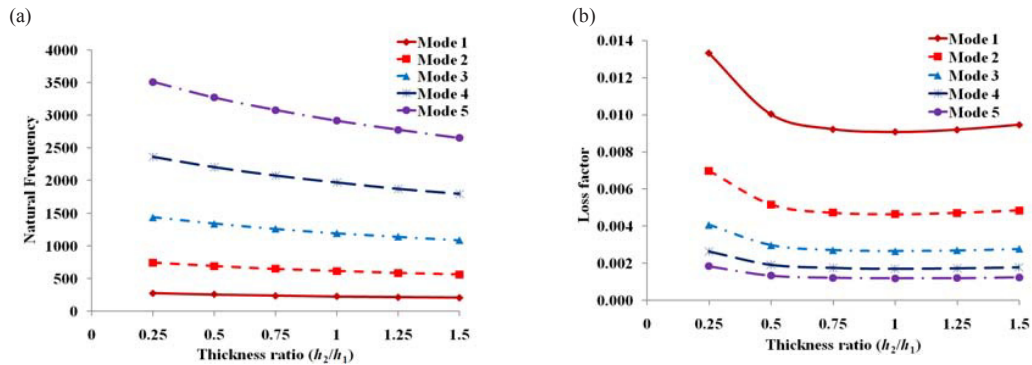


Fig.4. Influence of the thickness ratio (h_2/h_1) of the MR fluid layer at a magnetic field of 300 G under clamped-clamped end conditions (a) on the natural frequency and (b) on the loss factor

A comparison of the loss factor of the beam structures with different thickness of the MR layer at a magnetic field of 300 G for the clamped-clamped end conditions is also presented in Fig. 4 (b). The results show that significant in loss factor could be observed upto the ratio (h_2/h_1) of 0.75. When the thickness of the MR layer is increased, both the dissipated energy and the strain energy are increased. Hence, with increase in the thickness of the MR layer, the relative change in the dissipated energy would be considerably lower than that of the strain energy, which results in decrease in the loss factor. However, significant variation in loss factor could not be observed all the higher modes beyond the ratio (h_2/h_1) of 0.75.

4. Conclusions

In this study, the dynamic characterization of the MR fluid sandwich beam is performed considering the effect of shear deformation and rotary inertia effects of the elastic face layers. Mathematical modelling was developed using the finite element method by including the shear deformation and rotary inertia effects of the elastic layers. The controllable capabilities of MR fluid in a continuous elastic structure were investigated through conducting various parametric studies. It has been shown that the natural frequencies and loss factors for all the modes of the MR multi-layer beam could be increased by increasing the strength of the magnetic field. It has also been observed that the clamped-clamped and clamped free beams yield the highest and lowest natural frequencies, respectively, irrespective of the magnetic field. It is also shown that the loss factors decreases in higher modes compared to those of the fundamental mode. However, the similar trend could not be observed in camped-free end conditions, particularly in fundamental mode. It can be also concluded that the loss factors could be increased with increase in magnetic field at all the modes considered irrespective of the boundary conditions if the elastic layers are thick with inclusion of the shear deformation and rotary inertia effects. A decrease in the natural frequencies with increase in thickness of the MR layer is also observed. A significant variation in loss factors can be seen at fundamental modes compared to that of higher modes when the thickness of MR fluid is increased. It is also concluded that a significant variation in loss factor could not be observed at all the modes beyond a certain thickness ratio (h_2/h_1) of the MR fluid layer and the elastic layer. This shows that MR fluids can be effectively used in vibration control of multi-layer structures.

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