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Original article

Melting and viscous dissipation effects on MHD flow over a moving surface with constant heat source

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Abstract

The effects of viscous dissipation and heat source on MHD flow and heat transfer from a warm, electrically conducting fluid to melting surface parallel to a constant free stream are investigated numerically. This model constitutes highly non-linear governing equations which are transformed using similarity variables and are then solved by fourth order Runge–Kutta scheme along with shooting method. The influence of the various interesting parameters on the velocity and temperature fields within the boundary layer is discussed and explained graphically. It is noticed that the melting phenomenon rises the skin friction coefficient and declines the Nusselt number at the solid interface.

Keywords: Viscous dissipation; MHD; Boundary layer flow; Heat source; Moving surface

1. Introduction

MHD boundary layer flow and heat transfer accompanied by melting phenomenon have received recently considerable research attention in view of its engineering and industrial applications such as, material processing, crystal growth, latent heat storage, castings of metals, glass industry, and purification of materials. Epstein and Cho [1] investigated the melting heat transfer in the presence of laminar flow over a flat plate. Kazmierczak et al. [2,3] discussed the effects of convective flow on melting from a vertical plate. Gorla et al. [4] analysed the convection effect on melting from a vertical plate. Melting effects on mixed convective heat transfer flow from a vertical plate were investigated by Cheng and Lin [5–7]. Satya Narayana et al. [8] investigated the MHD nanofluid in a rotating system with radiation and heat source. Bakier et al. [9] studied melting effect on MHD flow past a vertical plate with porous medium. MHD heat transfer flow of a nanofluid over a stretching sheet was studied by Yohannes and Shankar [10].

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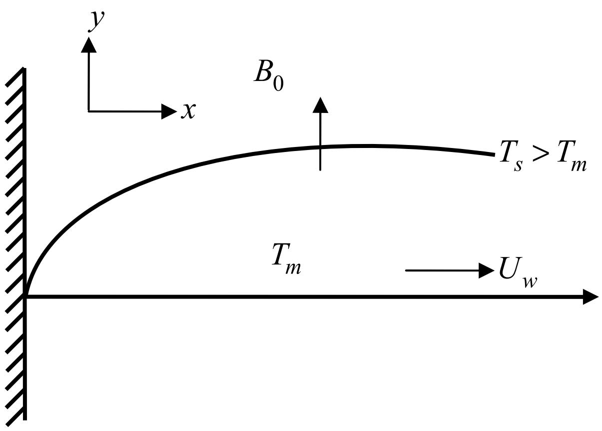

Fig. 1. Boundary layer on moving continuous surface.

Satya Narayana and Venkateswarlu [11] have analysed MHD nanofluid flow due to a vertical porous plate in a rotating system.

Heat transfer characteristics induced by a continuously moving surface are significant in industrial engineering processes such as in the polymer industry, lamination and melt spinning process. Ishak et al. [12] investigated the heat transfer flow over a melting and moving surface. Boundary layer performance on continuous stirring surface was first considered by Sakiadis [13,14]. Das [15] investigated melting effects on MHD flow over a melting and moving surface in the presence of radiation. Satya Narayana et al. [16] have studied MHD flow of micropolar fluid in a rotating system in the presence of Hall effect. The importance of heat transfer characteristics flow of a continuous stretching surface has been generated on this problem by [17–27].

The influence of viscous dissipation on heat transfer is important, especially for highly viscous flows even with moderate velocities. Viscous dissipation transforms the kinetic energy to internal energy (heating up the fluid) due to viscosity and hence increases the fluid motion. In view of this reason, various devices are designed in streambeds to reduce the kinetic energy of flowing water to reducing their erosive potential on banks and river bottoms. Due to this, the dimensionless parameter Eckert number is called the fluid motion controlling parameter. Motsumi and Makinde [28] investigated the flow of a nanofluid over a moving flat plate in the presence of radiation and viscous dissipation. Effects of thermal radiation and viscous dissipation on MHD flow of a nanofluid due to a stretching sheet were presented by Khan et al. [29]. Ferdowsn et al. [30] analysed the heat transfer flow of a nanofluids over a stretching sheet.

The main intention of the current investigation is to study the effects of viscous dissipation and thermal radiation on MHD flow from a warm, electrically conducting fluid to a melting surface with heat source. Numerical results are obtained by using a shooting technique together with Runge–Kutta scheme. The effects of various governing parameters on the velocity and temperature profiles as well as the skin friction coefficient and Nusselt number are explored through graphical and tabular forms.

2. Mathematical formulation

Consider two-dimensional MHD boundary layer flow of an incompressible electrically conducting fluid towards a moving surface melting at a steady rate into a warm liquid of the same material in the presence of heat source. The co-ordinate system and flow model of the problem is shown in Fig. 1. The flow is assumed to be in the x-direction which is taken along the moving surface and y-axis is normal to it.

In the present problem, our study is classified to the following assumptions:

(i) The surface is moving with a constant velocity U_w and the temperature of the melting surface is T_m.
(ii) The induced magnetic field is assumed to be uniform and is in the positive direction normal to the surface.

(iii) The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field.
(iv) The liquid phase far from the plate is maintained at constant temperature $T > T_m$.
(v) The temperature of the solid medium far from the interface $T_s < T$ is a constant.

Under the above assumption, the equations of motion and the equation representing temperature fields of the steady liquid flow obey the following boundary layer equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \vartheta\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}(u - U_\infty) \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_P}\frac{\partial q_r}{\partial y} - \frac{Q_H}{\rho C_P}(T - T_\infty) + \frac{\vartheta}{\rho C_P}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The suitable boundary conditions of the present problem are given as

$$\begin{aligned} &u = U_w(x) \quad T = T_m \quad for \quad y = 0 \\ &u \to U_\infty \quad T \to T_\infty \quad as \quad y \to \infty \end{aligned} \tag{4}$$

and

$$k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho\left\{\lambda + c_s(T_m - T_s)\right\} v(x, 0) \tag{5}$$

Using the Rosseland approximation, the radiative heat flux term q_r is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

We expand T^4 in Taylor's series and T_∞ and neglecting higher order terms, we get

$$T^4 = 4T\,T_\infty^3 - 3T_\infty^4 \tag{7}$$

Thus, we have

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

The momentum and energy equations (1)–(3) can be transformed into the corresponding ordinary differential equations by using the following similarity transformations:

$$\eta = \sqrt{\frac{U_\infty}{\nu x}}\,y \qquad \psi(x, y) = \sqrt{\nu x U_\infty}\,f(\eta) \qquad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m} \tag{9}$$

The stream function ψ is defined as follows:

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x} \tag{10}$$

Using Eq. (9) into Eqs. (2)–(3) can be rewritten as

$$f''' + \frac{1}{2}f\,f'' - M(f' - 1) = 0 \tag{11}$$

$$\frac{1}{Pr}(1 + R)\theta'' + \frac{1}{2}f\theta' - Q\,\theta + Ec\,f''^2 = 0 \tag{12}$$

The corresponding boundary conditions are

$$\begin{aligned} &f' = \varepsilon \quad \theta = 0 \quad Prf + H\theta' = 0 \quad at \quad \eta = 0 \\ &f' \to 1 \quad \theta \to 1 \quad as \quad \eta \to \infty \end{aligned} \tag{13}$$

where $\varepsilon = \frac{U_w}{U_\infty}$ is the moving parameter. It is worth mentioning that $\varepsilon = 0$ corresponds (Blasius flow) to the flow over a stationary surface caused by the free stream velocity. The case $0 < \varepsilon < 1$ is when the plate and fluid are moving in

Table 1
Comparison of Nusselt number of the present case with those of Das [15] for different values of $\varepsilon = 0.5$, $H = 1.0$, $K = 1.0$ and $M = 1.0$.

Pr	Nusselt number		R	Nusselt number	
	Das [15]	Present study $Ec = Q = 0$		Das [15]	Present study $Ec = Q = 0$
0.1	2.0833	2.083309	0.5	0.5356	0.539037
0.2	1.4586	1.453636	1.0	0.4405	0.444829
0.3	1.2521	1.258641	1.5	0.3389	0.338273
0.4	1.1461	1.146314	2.0	0.2534	0.254772

Table 2
Effect of fluid properties for different parameters on skin friction coefficient.

Skin friction coefficient					
ε	C_f	H	C_f	M	C_f
0.1	0.5035	0.5	2.4030	1.0	3.0551
0.2	0.5965	1.0	2.6470	2.0	3.0394
0.3	0.6912	1.5	2.7800	3.0	3.0237
0.4	0.7871	2.0	2.8650	4.0	3.0082

the same direction.

$$H = \frac{c_f(T_\infty - T_m)}{\lambda + c_s(T_m - T_s)} \quad M = \frac{\sigma x}{\rho U_\infty} B^2(x) \quad Ec = \frac{U_\infty^2}{\rho Cp(T_\infty - T_m)} \quad Q = \frac{Q^*}{\rho Cp U_\infty}$$

$$R = \frac{16\sigma^* T_\infty^3}{3kk^*} \quad Pr = \frac{\mu}{\alpha}$$

The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu, which are important for this kind of flow and they are defined as follows.

The local skin friction coefficient on the surface can be expressed as

$$C_f = \frac{\tau_w}{\rho u_e^2} = Re_x^{-1/2} f''(0) \tag{14}$$

The rate of heat transfer is usually expressed as the local Nusselt number, which is given by

$$Nu = -Re_x^{1/2}(1 + R)\theta'(0) \tag{15}$$

where $\tau_w = \mu \left\{ \frac{\partial u}{\partial y} \right\}_{y=0}$ is the wall shear stress and $Re_x = \frac{U_\infty x}{\nu}$ is the local Reynolds number.

Method of solution

The set of differential equations (9)–(10) are coupled and highly non-linear equations cannot be solved analytically. These equations are solved by using shooting method together with the fourth order Runge–Kutta method as discussed in Ref. [31].

3. Results and discussion

The influence of viscous dissipation on MHD boundary layer melting phenomenon with heat source on the velocity and temperature skin friction coefficient and local Nusselt number are analysed through graphs and tables. It is observed that, in the absence of viscous dissipation and heat source, the non-dimensional governing equations (9)–(10) with the corresponding boundary conditions (11) exactly coincide with equations (9) and (10) of Das [15] and also coincide with equations (7)–(9) of Ishak et al. [12] in the absence of M, Ec, R and Q values. The comparison of Nusselt number of the present case with those of Das [15] as well as Ishak et al. [12] for various values of $\varepsilon = 0.5$, $H = 1.0$ and $M = 1.0$ are shown in Table 1. It is clear that, in the absence of viscous dissipation and heat source, the results

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Table 3
Effect of fluid properties for different parameters on Nusselt number.

Nusselt number					
Ec	Nu	H	Nu	Q	Nu
0.2	0.6766	1.0	0.2217	1.0	0.0258
0.4	1.3774	1.5	0.2908	1.1	0.0423
0.6	2.0783	2.0	0.3649	1.2	0.0542
0.8	2.7790	2.5	0.4436	1.3	0.0626

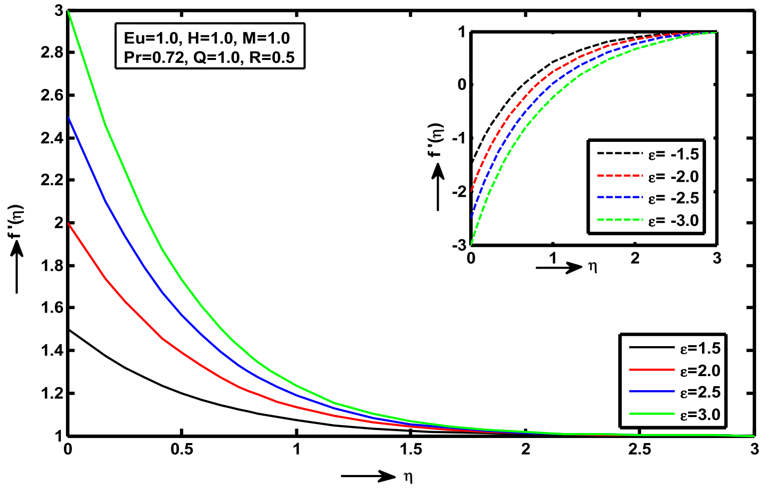

Fig. 2a. Velocity profiles for various values of ε.

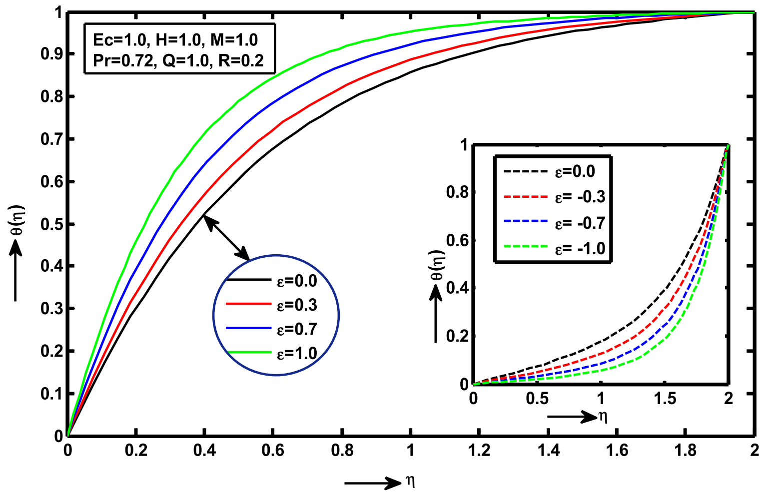

Fig. 2b. Temperature profiles for various values of ε.

agree very well with those of Das [15] and Ishak et al. [12]. The influence of ε, H and M on the skin friction coefficient is presented in Table 2. An increase in the values of moving parameter ε results in C_f enhances and the higher values of M the result reduced in C_f. Table 3 shows that the Nu increases for increasing values of Ec, H and Q.

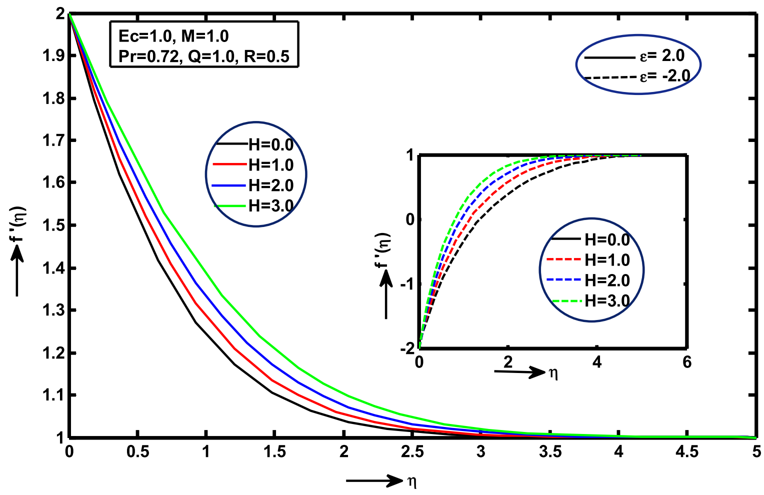

Fig. 3a. Velocity profiles for various values of H.

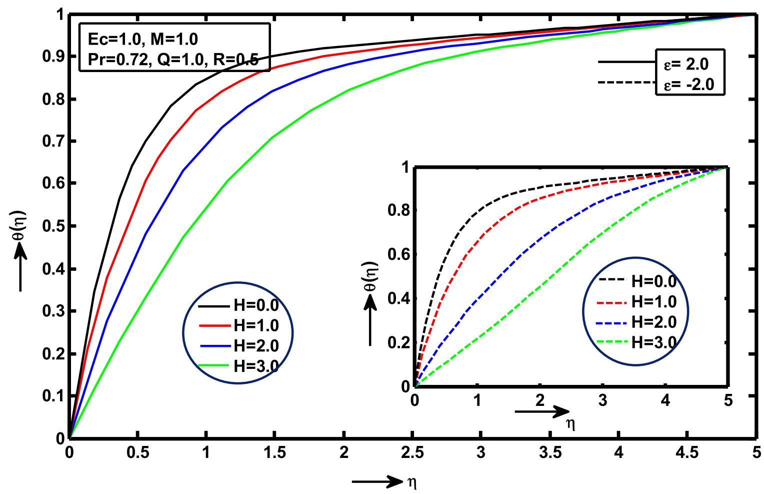

Fig. 3b. Temperature profiles for various values of H.

Figs. 2(a) and 2(b) depict the consequence of moving parameter ε on the velocity and temperature profiles. It is noticed that both the $f'(\eta)$ and $\theta(\eta)$ enhance with increasing the values of ε and therefore, the momentum boundary layer thickness increases in the flow region with increasing values of moving parameter. It is also observed that the $f'(\eta)$ and $\theta(\eta)$ show opposite behaviour when moving parameter $\varepsilon < 0$.

The effect of melting parameter H on the velocity and temperature distributions is shown in Figs. 3(a) and 3(b). It is observed that $f'(\eta)$ and $\theta(\eta)$ show opposite behaviour with increasing values of H within the boundary layer. This happens only for the moment of both the solid surface and free stream in the same directions. Also, it is noticed that the moving parameter $\varepsilon < 0$ has opposite behaviour on $f'(\eta)$ and no effect on $\theta(\eta)$. These results are same as noted in Ref. [15].

Fig. 4 illustrates the influence of the magnetic field parameter M on the velocity distribution. It is clear that the velocity distribution across the boundary layer reduces with an increase of M. Thus, the hydrodynamic boundary layer thickness decreases as M increases and also the fluid velocity decreases.

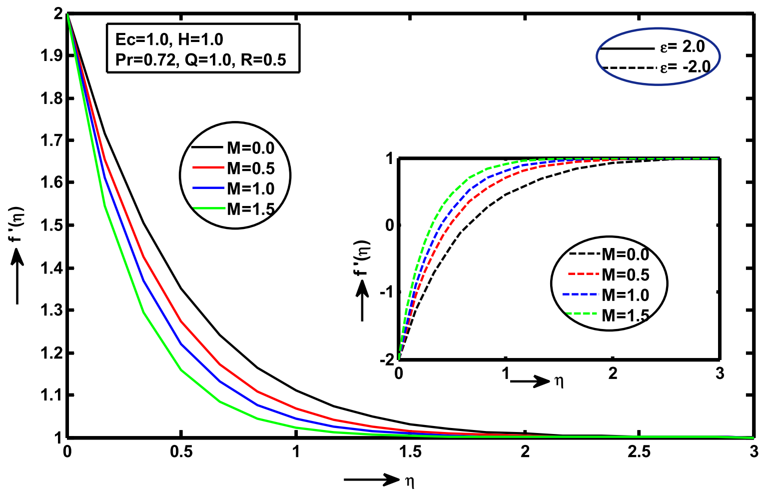

Fig. 4. Velocity profiles for various values of M.

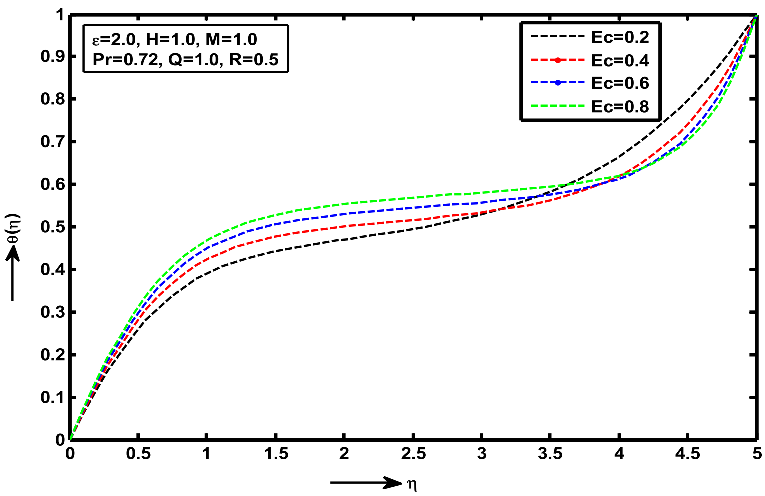

Fig. 5. Temperature profiles for various values of Ec.

Figs. 5 and 6 shows the temperature profile for dissimilar values of Eckert number Ec and Prandtl number Pr. It is observed that the temperature profile grows with rising values of Ec. Therefore, the effect of increasing Ec is to enhance $\theta(\eta)$ at any point in the thermal boundary layer. Further, the temperature is lowered with a rise in the Prandtl number and therefore the thermal boundary layer thickness reduced with rising values of Pr in the regime.

The effect of temperature profiles for various values of heat source parameter Q (i.e. $Q > 0$ heat source or $Q < 0$ heat sink) and radiation parameter R displayed in Figs. 7 and 8. It is observed that the temperature in the boundary layer region increases with $Q > 0$ and decreases for $Q < 0$. Further, the temperature decreases with increasing values of R.

Figs. 9 and 10 display the dissimilarity of C_f with magnetic field parameter M and moving parameter ε for various values of melting parameter H and magnetic field parameter M, respectively. It is observed that C_f enhances with escalating values of H and M. Thus, the surface friction force enhancing with an increase in the magnetic field strength is to impede the flow motion.

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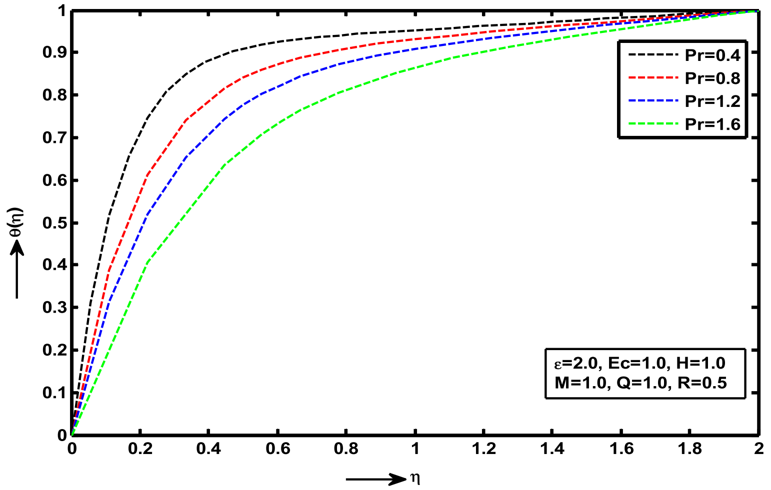

Fig. 6. Temperature profiles for various values of Pr.

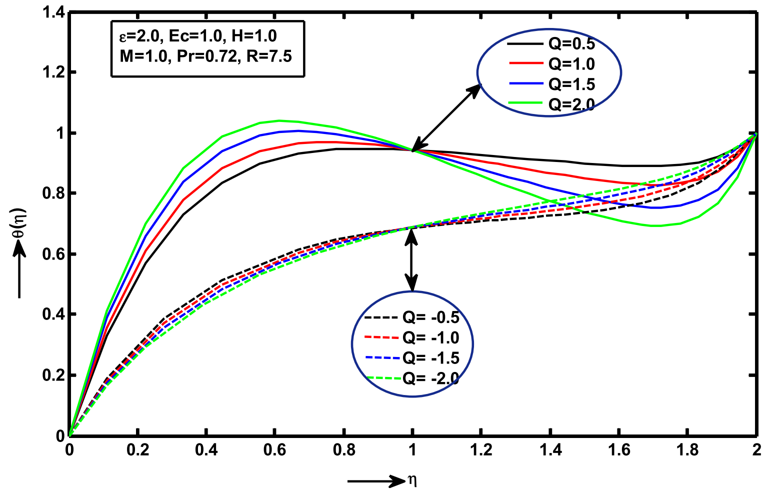

Fig. 7. Temperature profiles for various values of Q.

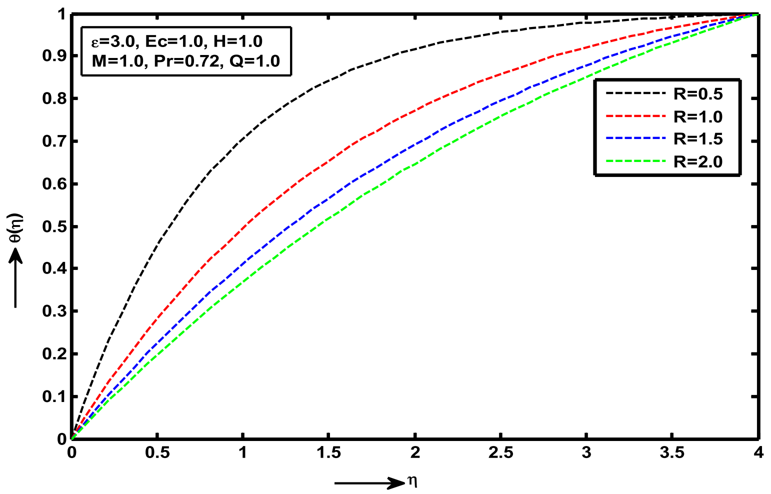

Fig. 8. Temperature profiles for various values of R.

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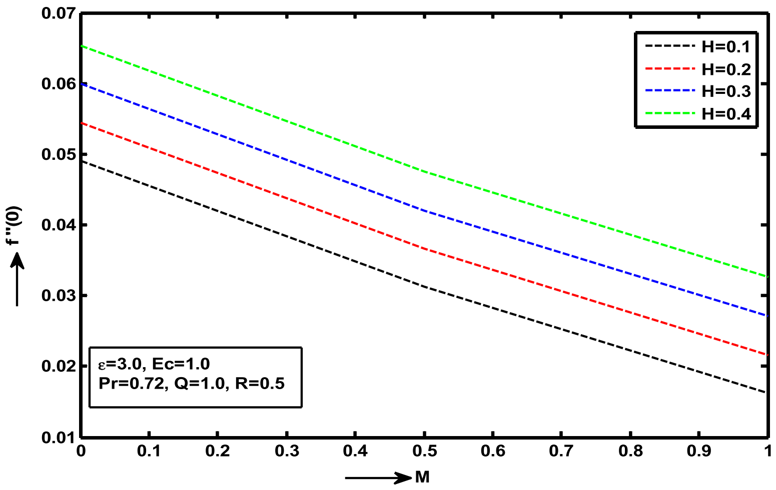

Fig. 9. Skin friction coefficient for various values of *H*.

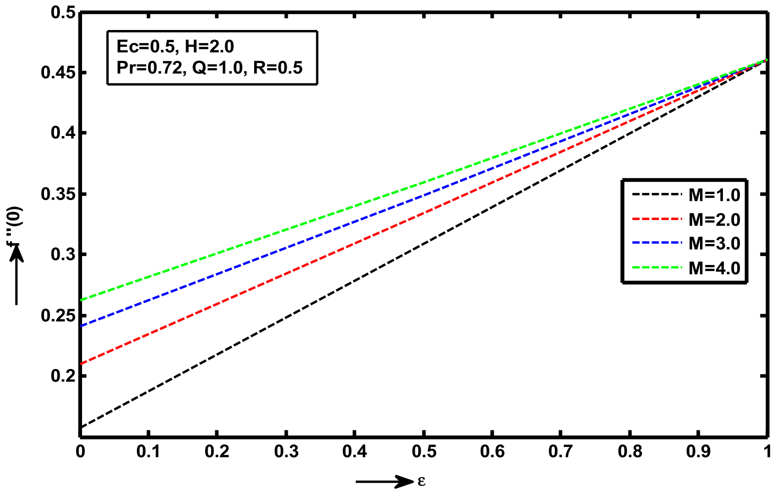

Fig. 10. Skin friction coefficient for various values of *M*.

Figs. 11–13 display the variation of Nusselt number Nu with radiation parameter R and Prandtl number Pr for various values of Eckert number Ec, heat source parameter Q and moving parameter ε, respectively. It is observed that Nu enhances with increasing values of Ec, Q and ε.

4. Conclusion

The influence of the various interesting parameters on the velocity and temperature fields as well as skin friction coefficient and Nusselt number are presented graphically. Apart from these, we made the following conclusions:

(i) The fluid velocity and temperature profiles enhance with rising values of ε.
(ii) The velocity increases as H increases and has the reverse effect on the temperature distribution.
(iii) The velocity reduces with an increase in the magnetic field parameter M.
(iv) The temperature increases with the increase of Ec, R and Q values.
(v) The skin friction and Nusselt number enhancing with increasing values of H, Ec and Q.

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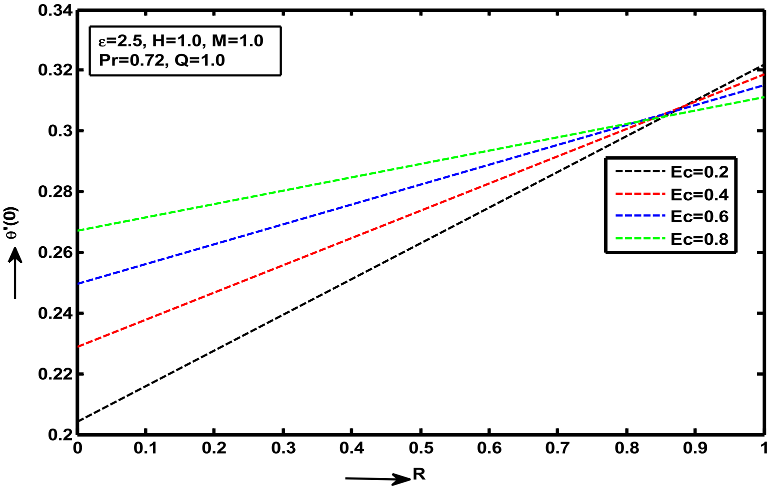

Fig. 11. Nusselt number for various values of Ec.

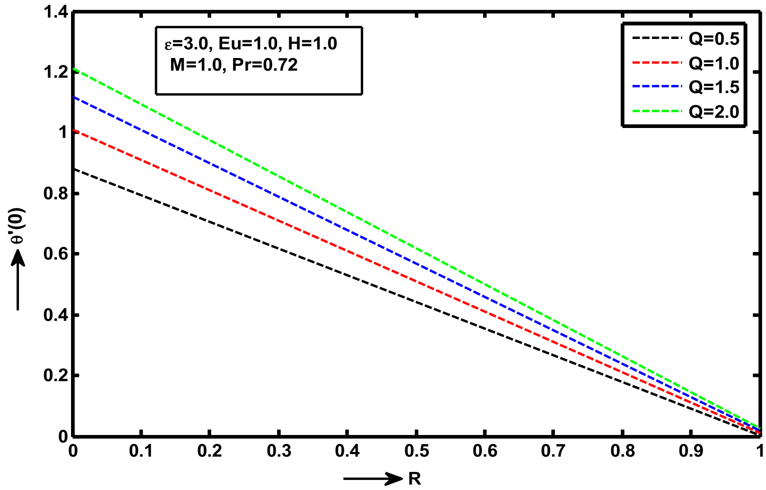

Fig. 12. Nusselt number for various values of Q.

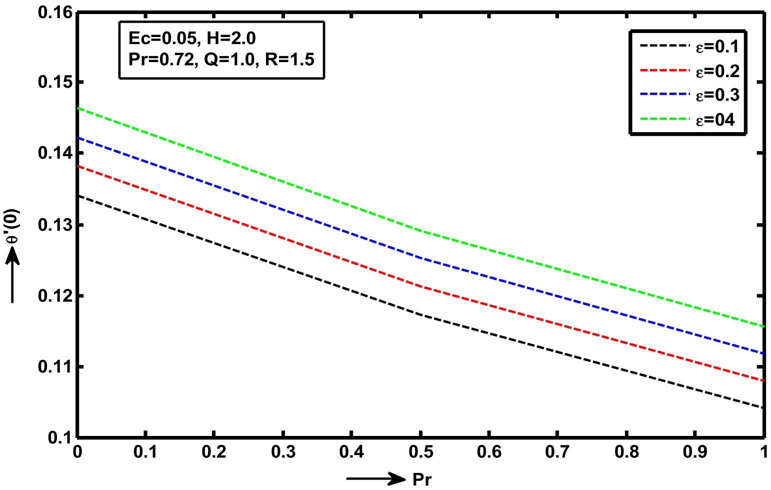

Fig. 13. Nusselt number for various values of ε.

Nomenclature

$B(x)$ applied magnetic field	T_∞ temperature far away from the plate
B_0 constant	U_w constant velocity
Cf skin friction coefficient	U_∞ free stream velocity
C_p specific heat at constant pressure	(u, v) velocity components
c_s heat capacity of the solid surface	(x, y) cartesian co-ordinates
Ec Eckert number	***Greek Symbols***
$f(\eta)$ dimensionless stream function	ε moving parameter
H dimensionless melting parameter	υ kinematic viscosity
k^* mean absorption coefficient	α thermal conductivity of the fluid
k thermal conductivity	ρ density of the fluid
M magnetic field parameter	σ electrical conductivity of the fluid
Nu Nusselt number	σ^* Stefan–Boltzmann constant
Pr Prandtl number	η similarity variable
Q heat source parameter	λ latent heat of the fluid
q_r radiative heat flux	μ coefficient of viscosity
R thermal radiation parameter	τ_w wall shear stress
Re_x Reynolds number	ψ stream function
T temperature of the fluid	$\theta(\eta)$ dimensionless temperature
T_m solid surface temperature	***Subscripts***
T_S solid surface temperature	w condition at the wall
	∞ condition at free stream

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