Wireless Pers Commun
DOI 10.1007/s11277-017-4882-z

On the Optimality of Multi-User Coordinated Zero-Forcing Beamforming Multicell Systems with Limited Feedback in Interference Channels

S. Balaji[1] · P. S. Mallick[1]

Abstract An efficient multicell coordinated zero-forcing channel feedback scheme is proposed in this paper. The objective of the proposed feedback design is to control the rate of the user by adaptive allocation of feedback bits of each user as a function of individual channel status thereby keeping the total feedback budge constant. The proposed feedback allocation is studied in interfering broadcast channel (IFBC) and also in time varying channels (TVC) with feedback update duration. First the individual feedback rates of user are derived using convex optimization thereby presenting low complexity algorithm to optimize the channel feedback between inter-user interference and inter-cell interference. The rate offset arising from channel quantization in both IFBC and TVC is investigated by employing the coordinated zero forcing beamforming. Moreover, closed form expressions are derived for necessary feedback scaling and frequency of update duration to achieve the multiplexing gain and throughput. Finally, numerical evaluation results show that the proposed channel feedback schemes outperforms other conventional schemes and assist in base station cooperation management. It can be concluded that frequent update of channel state information and suppression of dominant interferences can yield significant improvements in the sumrate performance of limited feedbback systems.

Keywords Limited feedback · Interference channels · Time varying channels · Multiplexing gain · Coordinated zero-forcing beamforming · Feedback update

✉ S. Balaji
sbalaji@vit.ac.in

P. S. Mallick
psmallick@vit.ac.in

[1] School of Electrical Engineering, VIT University, Vellore 632014, India

1 Introduction

There has been substantial research on multi-user multiple input multiple output (MIMO) schemes because of their promising role in next generation cellular networks. Multi-antenna cellular networks are most affected by inter-user and inter-cell interference. The downlink interference cancellation schemes are extensively analysed in the recent past with limited backhaul feedback. The limited feedback in multi-antenna techniques in downlink environment increases the overall throughput. The throughput improvement is degraded not only by inter-user interference (IUI) but also by inter-cell interference (ICI). The overall throughput improvement performance depends on the suppression of the above two types of interference. Downlink transmission utilizing zero-forcing precoding in conjunction with finite rate feedback system has been studied [1]. The sum rate performance with finite rate feedback in multi-antenna system is analysed. Expressions and tradeoffs between number of feedback bits, the number of users, and the signal to noise ratio (SNR) are observed [2]. The performance of multi-antenna systems in downlink under random vector quantization (RVQ) is also analysed [3]. In order to suppress the interference, lot of beamforming strategies are proposed in the past. [4–6].

In addition to beamforming, adaptive bit partitioning strategy has also been proposed by many researchers. In the recent literature, with random vector quantization, expressions are derived for bit partitioning. In almost all the models, the stronger channels are quantized with more bits and smaller delays but weaker channels are allocated with fewer bits. The average loss (offset) in throughput with adaptive bit partitioning is quantified in time varying channels (TVC) when the desired signals of users are exchanged in the system with delay [7]. In [8] adaptive bit partitioning strategy for extended Clarke's model has been introduced to quantize the channel state information of channel with component fluctuations. A measure of trade-off between performance bottleneck and bit allocation are explored and it is shown that the bit allocation is skewed highly when the channels are closely correlated [9]. Sum rate has also been characterised using differential feedback scheme in temporally correlated channels and two stage feedbacks depending on individual channel status is also presented [10, 11]. Moreover MIMO single cell and multi-cell coordinated limited feedback transmissions are proposed by many authors and expressions for bit allocation and system performance are also presented.

The optimized bit allocation and feedback rate with minimum total power transmission subject to signal to interference noise ratio (SINR) constraint is also discussed in [12]. Analytical assessment of coordination using stochastic geometry for interference cancellation by suitably allocating feedback bits is also provided [13]. Rate loss is characterised by considering coordinated beamforming in MIMO systems. In these MIMO systems expressions are derived for sum rate and number of feedback bits [14–16]. The rate loss due to channel feedback update period is also presented [17]. Consequently, the cooperation among users or Base Stations (BSs) is utilized to enhance the performance of systems under study that have very high levels of interference which are necessarily arising from inter-user and or inter-cell. The capabilities of backhaul link limit the range of coordination.

The basic Question is that, is it necessary to always allocate feedback bits to the interfering base stations even if the inter-cell interference (ICI) is bare minimum? The second question to be answered is that is the ICI significant in all the regions of a cell to cause throughput degradation? To the best our knowledge all most all of the adaptive feedback allocations, feedback resources are allocated based on their signal strengths. Moreover, in most of the previous works either they characterized the MIMO systems for interfering broadcast channels (IFBC) by assuming channel is time invariant during feedback period and feedbacks are allocated or MIMO feedback is studied with channel temporal correlation in time varying channels (TVC). In both IFBC and TVC, the volume of feedback is measured with complex interference management and sent only after channel information is updated. In almost all of these models, they have not quantified either amount of feedback scaling or performance improvements in-terms of inter-user and or inter-cell interference in varying channel conditions. Moreover, these recent studies do not characterize multiplexing gain in-order to achieve the required feedback rate in time varying channels i.e. channels with temporal correlations taken into account to model the delay. This shortcoming motivates us to derive the required mathematical expressions to characterize the performance and to allocate the feedback between serving BS and interfering BSs based on whether the interference is significant to cause throughput degradation. Moreover, in [18], the authors proposed bit partitioning using interference grading threshold. Interference mitigation through inter-cell interference coordination using effective SINR metric is investigated in [19]. Feedback optimization in a two cell coordinated beamforming MIMO system to maximize the SINR is also described [20]. These reccent works failed to categorize the dominant interferers. Moreover, if the interference is weak, treating the interference as noise can be an optimal approach [21]. In [22], it is proved that TVC requires much higher feeback allocation to achieve the required sum rate as that of IFBC. To curtail the amount of backhaul feedback bits, and to find which interference is stronger to cause throughput degradation, a bit partitioning scheme with interference grading is proposed in this paper. The main objective of this paper is to maximize the channel feedback efficiency and for a total feedback bits per user, a algorithm is developed to optimally allocate bits between serving base station (BS) and interfering base stations with feedback update duration. The key idea is to keep interference grading threshold ε_T to study the system performance improvements in IFBC, TVC and closing the SNR gap to achieve multiplexing gain. In the proposed convex optimization feedback allocation, we derived the feedback bits as a function of grading parameter (ratio of power of IUI and ICI), residual feedback and feedback update duration. To keep a constant rate off-set in IFBC and TVC, the total feedback is scaled and the scaled feedback is now a function of grading parameter ε in addition to antennas and number of Users. The proposed method extend desired base station constant feedback region (Non-Cooperative Region) thereby reducing the limited feedback coordination overhead. Finally, the performance of feedback scaling in IFBC and TVC is demonstrated by simultaneously varying feedback update duration and SNR.

The rest of the paper is organized as follows. In Sect. 2, the system model with the methodology of feedback and coordinated beamforming is described. Section 3 describes the proposed IFBC adaptive finite rate bit allocation scheme and characterization of rate loss for IFBC, and Sect. 4, extended the proposed methodology to TVC. The theoretical results are corroborated with analytical simulations in Sect. 5, and finally conclusions are given in Sect. 6.

2 System Model

The system consists of K-Cell Multiple Input Single Output (MISO) with finite rate feedback shown in the Fig. 1. Each cell in the MISO system containing one BS i.e. BS_i, where $i = 1, 2, \ldots, K$, and the BSs are equipped with M antennas. Each BS sends L data streams to the L users of interest. The user l at the ith BS is denoted (l, i). Assuming equal

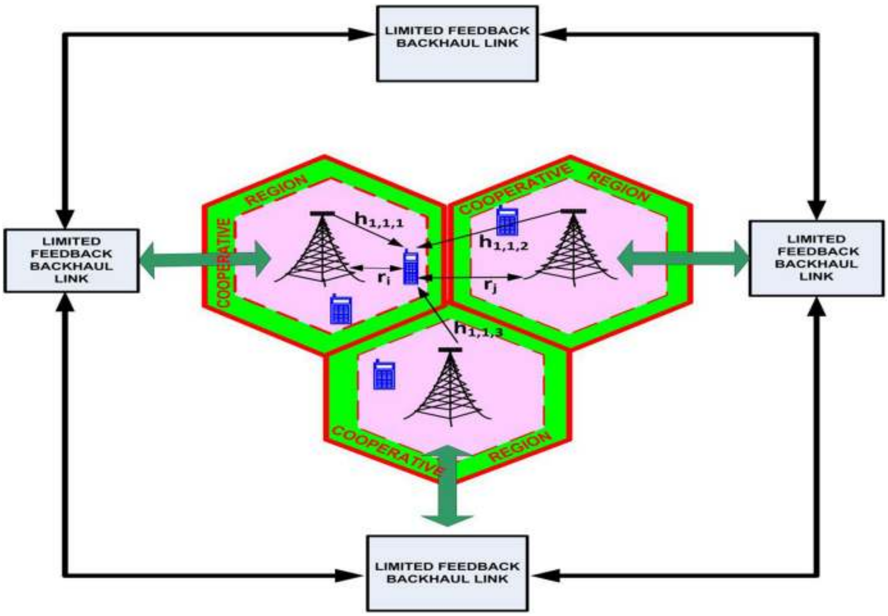

Fig. 1 Limited feedback MIMO system

power allocation over L users in the BS, the received signal at the user (l, i) is given by [14]

$$y_{l,i} = \sqrt{\gamma_{l,i,i}} h_{l,i,i}^{H} w_{l,i} s_{l,i} + \underbrace{\sum_{m=1,m\neq l}^{L} \sqrt{\gamma_{l,i,i}} h_{l,i,m}^{H} w_{m,i} s_{m,i}}_{IUI} + \underbrace{\sum_{j=1,j\neq i}^{K} \sqrt{\gamma_{l,i,j}} h_{l,i,j}^{H} \sum_{l=1}^{L} w_{l,j} s_{l,j}}_{ICI} + n_{l,i}, \quad (1)$$

where $\gamma_{l,i,j}$ is the interference power received by the user (l, i), $h_{l,i,j}^{H}$ is the channel vector of size $M \times 1$ from $BS_j, j = 1, 2, \ldots, K, j \neq i$. The vector $w_{l,i}$ stands for the beamforming vectors for user (l, i) with size of $M \times 1$ and having constraints on normalization, i.e. $\|w_{l,i}\| = 1$. The parameter $s_{l,i}$ denotes the data symbol for the user and $E(|s_{l,i}|^2) = 1$. All the channel elements are independent identically distributed complex random variables having zero mean and unit variance. The scalar $n_{l,i}$, denotes the additive white Gaussian noise at the receiver with a variance of one. The path loss incurred by the user from the ith BS is given by$(1 + r_i)^{-\alpha}$, where r_i is the distance between the Mobile Station (MS)/ user of interest to the ith BS and α is the path loss exponent. The exponential path loss attenuation model is considered to study the system with high accuracy even at a lower coverage distance from BSs.

The system described in above Eq. (1), is studied in this work under two proposed Channel Models. The system under Interfering Broadcast Channel (IFBC) is analysed by assuming the channel is time invariant. In Time Varying Channel Model (TVC), channel temporal correlation is modelled using Gauss Markov model to incorporate the time varying behaviour. In both of these models, feedback bits are optimized by taking interference grading threshold as one of the parameters. To keep a constant rate offset, closed form expressions are derived and results are analysed with feedback update duration. The interference grading threshold is to choose the adaptive feedback allocation for IUI and ICI and it defines the region of BS cooperation thereby significantly increasing the cell average sum rate. The cell average performance in the proposed model is characterized in Non-Cooperative region (IUI dominates compared to ICI) and in Cooperative region (ICI dominates compared to IUI) and the regions are indicated in Fig. 1.

In the proposed system, Random Vector Quantization (RVQ) is applied to quantize the channel distribution information (CDI). In the feedback analysis, channel quality information (CQI) feedback is not included in total feedback calculation and each user satisfies the total feedback constraint i.e. $Ka^{CQI} + \sum_{l=1}^{L} b_l = B_{Total}$, where a^{CQI} is the feedback bits for CQI of each user. Since both quantized CDI of serving and interfering users are known, each BS constructs the beamforming vectors $w_{l,i}$ given in Eq. (1) in such a way that the inter user interference (IUI) and Inter carrier interference (ICI) becomes zero. In finite rate feedback system, IUI and ICI are not fully eliminated and as a result of partial elimination, there is a residual interference which deteriorates the achievable rate of the system. To represent the residual interference of finite rate feedback system, the channel $\tilde{h}_{l,i,j}$ is decomposed by the two orthogonal basis by using quantized CDI Information as $\tilde{h}_{l,i,i} = \hat{h}_{l,i,j}(\cos\theta_{l,i,j}) + q_{l,i,j}(\sin\theta_{l,i,j})$, where $\theta_{l,i,j}$ denotes the angle between real and quantized channel direction vectors. The quantity $q_{l,i,j}$ represents the error vector due to channel quantization. Since the beamforming vector is designed to null out the IUI and ICI, the rate of the user with residual interference is written as

$$R_{l,i}^{FB} = \log_2\left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^H w_{l,i}\right|^2}{1 + \tilde{I}_{IUI} + \tilde{I}_{ICI}}\right) \tag{2}$$

where $\tilde{I}_{IUI} = E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2 \sin^2\theta_{l,i,i}\sum_{m=1,m\neq l}^{L}\left|q_{m,i,i}^H w_{m,i}\right|^2$ and $\tilde{I}_{ICI} = \sum_{j=1,j\neq i}^{K} E_{l,i,j}$ $(1+r_j)^{-\alpha}\left\|h_{l,i,j}\right\|^2\sum_{l=1}^{L}\sin^2\theta_{l,i,j}\left|q_{l,i,j}^H w_{l,j}\right|^2$. In the above equation, we have substituted the desired signal of the user $\gamma_{l,i,i}$ in-terms of path loss as $\gamma_{l,i,i} = \frac{E_{l,i,i}}{(1+r_i)^{\alpha}}$ where $E_{l,i,i}$ is the transmitted signal power from the ith base station. The interference signal power is measured as $\gamma_{l,i,j} = \frac{E_{l,i,j}}{(1+r_j)^{\alpha}}$, and the Gaussian noise is assumed to be having unit variance.

3 Finite Rate Feedback Bit Partitioning in IFBC

The rate loss $\Delta R_{l,i}$ in interfering broadcast channel is the difference between achievable rate at the user with perfect CSI and limited feedback CSI i.e. $\Delta R_{l,i} = E(R_{l,i}^{PCSI} - R_{l,i}^{FB})$where $R_{l,i}$ is the rate achievable by the user when perfect CSI is available. The rate $R_{l,i}^{PCSI}$ is given as

$$R_{l,i}^{PCSI} = \log_2\left(1 + \gamma_{l,i,i}\left|h_{l,i,i}^H w_{l,i}^P\right|^2\right) \tag{3}$$

where $w_{l,i}^P$ is the beamforming vector when perfect CSI is available at the BS_i. The average sum rate of the user with quantized CDI (limited Feedback) using Coordinated zero forcing beamforming is expressed as

$$E\left[R_{l,i}^{FB}\right] = E\left\{\log_2\left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^H w_{l,i}\right|^2}{\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1}\right)\right\} \tag{4}$$

The average sum rate of the user with quantized CDI (limited Feedback) using coordinated zero forcing beamforming expressed in Eq. (4) is function of residual inter-cell and inter-user interference. To categorize dominant interference and to identify the cooperation region, one of the interferences to be modelled with respect to the other. The inter-cell interference is characterized by a single parameter, i.e. a grading factor ε belongs to the interval [0, 1) which is now defined as inter-cell interference power to the inter-user interference power, i.e.,

$$\varepsilon = \frac{ICI}{IUI}, \quad \varepsilon = \frac{\sum_{j=1,j\neq i}^{K} E_{l,i,j}(1+r_j)^{-\alpha}\left\|h_{l,i,j}\right\|^2 \sum_{l=1}^{L} \sin^2\theta_{l,i,j}\left|q_{l,i,j}^H w_{l,j}\right|^2}{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2 \sin^2\theta_{l,i,i}\sum_{m=1,m\neq l}^{L}\left|q_{m,i,i}^H w_{m,i}\right|^2} \tag{5}$$

This grading of residual interference is very much necessary since if the user in the middle of the cell i.e. in the non-cooperative region, the user is more affected by IUI rather than ICI. In the non-cooperative region allocating more bits to the residual ICI which is in the order of approximately 0.1–2% of IUI is waste of system resources and moreover the users will also not be supported the guaranteed throughput because of unnecessary allocation of limited feedback bits. In-order to solve this problem of wasteful allocation of limited resources and not to degrade the average throughput, grading of interference is proposed in this work. In this grading of interference, the stronger interference in the region is graded suitably and is allocated more bits based the value the interference against the grading threshold ε_T. Once the graded interference is greater than or equal to the threshold, the bits are allocated suitably by the proposed algorithm defined in end of this section.

But at the same time if the user is in the cooperative region, ICI will have more pronounced effect than IUI. For example if ICI equals 50% of IUI or higher (ICI will almost be equal to IUI at the cell edge i.e. 100%) sufficient number of bits as that of IUI is allocated to null out the residual interference caused by ICI. That is, the ICI is significant to reduce the throughput and in such a cases the algorithm compares the grading threshold against the ε_{sig}, the significant threshold and allocates the bits suitably between the IUI and ICI. By substituting the value of ICI, i.e. ICI (IUI $\times$ ε) and ICI in the Eq. (4), the expected rate becomes

$$E\left[R_{l,i}^{FB}\right] = \left\{\log_2\left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^H w_{l,i}\right|^2}{(1+\varepsilon)\left(E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2 \sin^2\theta_{l,i,i}\sum_{m=1,m\neq l}^{L}\left|q_{m,i,i}^H w_{m,i}\right|^2\right) + 1}\right)\right\} \tag{6}$$

Now using the fact that the random variables $\|h_{l,i,j}\|^2$, $\sin{}^2\theta_{l,i,j}$ and $|q_{l,i,j}^H w_{l,j}|^2$ are linearly independent each other. Using the upper bounds of quantization error [3] i.e. $E(\sin^2\theta_{l,i,i}) \prec 2^{-\frac{b_l}{M-1}}$ and taking $\sum{}_{l=1}^{L} b_l = B_{res}$, where B_{res} is the total bits allocated for both IUI and ICI interference. Thus the average sum rate when there are L users from the above Eq. (6) is written as

$$E\left[R_{l,i}^{FB}\right] \cong E\left\{\sum_{l=1}^{L} \log_2\left(\frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^{H} w_{l,i}\right|^2}{1+\left[E_{l,i,j}(1+\varepsilon)(1+r_i)^{-\alpha}\left(\frac{M}{M-1}\right)\right]2^{-\frac{b_l}{M-1}}}\right)\right\} \tag{7}$$

For notational brevity, let us define $\delta_l = E_{l,i,i}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^{H} w_{l,i}\right|^2$ and $E_{l,i,j}(1+\varepsilon)(1+r_i)^{-\alpha}$ $\left(\frac{M}{M-1}\right) = P_l$. The channel feedback bit allocation is now formulated to maximize rate with respect to b_l and $b_l \le b_{\max}$. The $b_{\max}$ constraint for the non-negative integer b_l is to restrict the total number of feedback bits for CDI quantization is less than or equal to B_{Total} - Ka^{CQI}. Now to obtain a solution with minimum complexity, a suboptimal approach from convex optimization is provided with continuous relaxation techniques [11]. Now the problem is re-written as

$$\begin{aligned} &f(b) = -\sum_{l=1}^{L} \log_2\left(\frac{(1+r_i)^{-\alpha}\delta_l}{1+P_l 2^{-\frac{b_l}{M-1}}}\right) \\ &\text{Subject to } 1^T.b^l - B_{res} = 0 \\ &-b^l \le 0 \end{aligned} \tag{8}$$

where B_{res} is the number of feedback bits essentially reserved for CDI (IUI and ICI) quantization after allocating to CQI Quantization. This B_{res} plays a vital role in determining the rate of user. Thus the above problem is the convex optimization problem and can be solved by Lagrange dual optimization method [23]. The Lagrangian dual function for the above problem is

$$L(\lambda, \upsilon) = \inf_{b^j \in D}\left\{-\sum_{l=1}^{L} \log_2\left(\frac{(1+r_i)^{-\alpha}\delta_l}{1+P_l 2^{-\frac{b_l}{M-1}}}\right) + \sum_{l=1}^{L} \lambda_l(-b^l) + \upsilon(1^T.b^l - B_{res})\right\} \tag{9}$$

The Karush–Kuhn–Tucker (KKT) conditions for the above lagrangian dual problem are

$$\begin{aligned} &-b^{l*} \le 0,\ (Primal) \\ &1^T b^{l*} - B_{res} = 0,\ (Primal) \\ &\lambda_l^* \ge 0,\ (Dual) \\ &\lambda_l^* b^{l*} = 0,\ (Complementary\, slackness) \\ &-\frac{P_l}{(M-1)\left[2^{\frac{b_l}{M-1}} + P_l\right]} - \lambda_l^* + \upsilon^* = 0,\ (Gradient\, of\, Lagrangian) \end{aligned} \tag{10}$$

The problem appears to be convex and Slater's condition is satisfied, the Karush–Kuhn–Tucker Condition yields the optimal solution. By utilizing the slack parameters λ_l^* and υ^*, the solution to the problem found to be

$$b_l = (M-1)\left[\log_2\left\{(P_l)\left(\frac{1}{(M-1)\upsilon} - 1\right)\right\}\right] \tag{11}$$

and the sum of all b_l, that is $\sum_{l=1}^{L} b_l = B_{res}$. Substituting the value of P_l, the above Eq. (11) becomes

$$b_l = (M-1)\left[\log_2\left\{\left[E_{l,i,j}(1+\varepsilon)(1+r_i)^{-\alpha}\left(\frac{M}{M-1}\right)\right]\left(\frac{1}{(M-1)\upsilon} - 1\right)\right\}\right] \tag{12}$$

where υ is found by water filling algorithm. Assume all b_l is greater than zero ($b_l > 0$), and utilizing the fact that the sum $\sum_{l=1}^{L} b_l = B_{res}$, the value of b_l found using water filling algorithm from Eq. (11) is

$$\upsilon = \left(\frac{1}{(M-1)}\right)\left\{\frac{\left(\prod_l^L P_l\right)^{\frac{1}{L}}}{\left(2^{\frac{B_{res}}{M-1}}\right)^{\frac{1}{L}} + \left(\prod_l^L P_l\right)^{\frac{1}{L}}}\right\} \tag{13}$$

To find the value of B_{res}, an iterative algorithm that evaluates B_{res} in Eq. (13) starting with the assignment of interference grading threshold ε_T is proposed. The algorithm is stated as follows.

Alogrithm:

Require: Define the grading threshold $\varepsilon_T, \varepsilon_{sig}$.

1. for all MSs $l = 1, 2, \ldots, L$ do
2. Initialize the grading parameter $\varepsilon_{l,i,j} = 0$ all r_i and r_j.
3. Find $\gamma_{l,i,i}$ for all l where $l \neq m$
4. Calculate IUI of the user (l, i) for all r_i
5. for all BSs $j = 1, 2, \ldots, K, j \neq i$, find $\gamma_{l,i,j}$ for all BSs j
6. Calculate ICI of the user (l, i) for all r_j
7. Initialize $B_{res} = b_{\max}$ for all r_i
8. **For** all r_i, r_j, find $\varepsilon = \frac{ICI}{IUI}$, **End**
9. **If** ($\varepsilon \geq \varepsilon_T$ and $\varepsilon < \varepsilon_{sig}$) then
10. Calculate $B_{res} = \left\lceil b_{\max}\left(\frac{1}{1+\varepsilon}\right)\right\rceil$ for all r_i and r_j.
11. **else If (** $\varepsilon \geq \varepsilon_{sig}$ **)** then
12. Calculate $B_{res} = \left\lfloor \frac{b_{\max}}{2} + (M-1)abs\left[\log_2\left(\frac{1}{\varepsilon}\right)\right]\right\rfloor$ for all remaining r_i, r_j till $r_i = r_j$.
13. **End**

Comments on Operation of Algorithm

For all the users in the ith cell, the grading factor ε is calculated for all r_i, r_j. If the grading factor is less than the grading threshold ε_T, the serving BS gets $b_{\max}$ bits and the interfering base station will not be assigned any bits. If the grading factor lies greater than or equal to the grading threshold but less than the significant threshold ε_{sig}, the ICI is significant to cause interference and deteriorates the sum-rate. The proposed algorithm starts allocating bits to the desired base stations using step 10 of the algorithm on successive r_i till it reaches $r_i = r_j$. The remaning bits $\mathrm{b}_{\max} - \mathrm{B}_{\mathrm{res}}$ are allocated to the interferring BSs. But at the same time if the grading factor ε is greater than ε_{sig}, the region belongs to region of ICI (i.e. at the cell edge) and the B_{res} is calculated based on the step 12 of the algorithm. The desired base station gets B_{res} and successively calculated till $r_i = r_j$. The interfering base station is allocated the remaining bits.

3.1 Minimization of Rate Offset in IFBC

Quantification of number of bits required in limited feedback system is necessary to characterize the power offset and rate loss. To study the number of feedback bits required to have a constant rate loss of Ω Bits per second/Hertz (bps/hz) compared to the perfect CSI rate in interfering broadcast channel (IFBC), the scaling of law of bits required for the user to maintain the rate is derived in this section. The expected rate offset of proposed IFBC with limited feedback perfect CSI is given as $\Delta R_{l,i} = E[R_{l,i}^{PCSI} - R_{l,i}^{(PFB)}]$. The rate offset can be obtained by subtracting the Eq. (6) from Eq. (3) i.e.

$$\Delta R_{l,i} = E\left\{ \log_2\left(1 + \gamma_{l,i,j}\left|h_{l,i,i}^H w_{l,i}^P\right|^2\right) - \log_2\left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^H w_{l,i}\right|^2}{\underbrace{\left(E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2 \sin^2\theta_{l,i,i}\sum_{m=1,m\neq l}^{L}\left|q_{m,i,i}^H w_{m,i}\right|^2\right) + \left(\varepsilon\left(E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2 \sin^2\theta_{l,i,i}\sum_{m=1,m\neq l}^{L}\left|q_{m,i,i}^H w_{m,i}\right|^2\right)\right) + 1}_{res=\tilde{I}_{IUI}+\tilde{I}_{ICI}+1}}\right)\right\} \tag{14}$$

By applying Jensen’s inequality, the rate loss can now be written as

$$\begin{aligned} \Delta R_{l,i} = {} & E\left[\log_2\left(1 + E_{l,i,i}(1+r_i)^{-\alpha}\left|h_{l,i,i}^H w_{l,i}^P\right|^2\right)\right] \\ & - E\left[\log_2\left\{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\left|\tilde{h}_{l,i,i}^H w_{l,i}\right|^2\right\}\right] \\ & + E\left[\log_2\left\{\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1\right\}\right] \end{aligned} \tag{15}$$

The log function is monotonically increasing and considering the fact that $w_{l,i}^P$ and $w_{l,i}$ are independent and isotropically distributed, the rate loss reduces to the expected value of residual IUI and ICI interference. Thus the rate loss of the above Eq. (15) reduces to $\Delta R_{l,i} = E\left[\log_2\left\{\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1\right\}\right]$. After algebraic manipulation, the value of $\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1$ from the Eq. (14) is $\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1 = \Big((1+\varepsilon)\Big(E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}\right\|^2\sin^2\theta_{l,i,i}\sum_{m=1,m\neq l}^{L}$ $\left|q_{m,i,i}^H w_{m,i}\right|^2\Big) + 1\Big)$. By adapting the similar procedure used for deriving Eq. (7) from Eq. (6), the residual interference is calculated as $\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1 = E_{l,i,j}(1+\varepsilon)(1+r_i)^{-\alpha}$ $\left(\frac{M}{M-1}\right)2^{-\frac{b_l}{M-1}}$. Since there are L users in the system and considering each user rate loss, the rate loss is now upper bounded by

$$\Delta R_{l,i} \leq \log_2\left(1 + \left(E_{l,i,j}(1+\varepsilon)(1+r_i)^{-\alpha}\left(\frac{M}{M-1}\right)2^{-\frac{b_l}{M-1}}\right)\right) \tag{16}$$

The rate to be maintained within a $\log_2$ (Ω) bps/Hz. By cancelling the logarithm on both sides and after substituting the value of b_l from the Eq. (12), the rate offset is bounded to

$$(\Omega - 1) \geq 2^{-\frac{B_{res}}{L(M-1)}} \left(\prod_{l=1}^{L} E_{l,i,j} (1+\varepsilon)(1+r_i)^{-\alpha} \left(\frac{M}{M-1} \right) \right)^{\frac{1}{L}} \tag{17}$$

The above equation is rearranged to get total feedback bits B_{res} as

$$B_{res} = (M-1)(L) \left[\log_2 \left(\frac{1}{(\Omega - 1)} \right) + \log_2 \left(\left(\prod_{l=1}^{L} E_{l,i,j} (1+\varepsilon)(1+r_i)^{-\alpha} \left(\frac{M}{M-1} \right) \right)^{\frac{1}{L}} \right) \right] \tag{18}$$

The B_{res} is a function of grading factor ε, the number of users L and antennas M. To maintain a constant rate loss, the B_{res} needs to be scaled appropriately. The effect of scaling to get fixed power offset for different ε is illustrated with example in numerical simulation section.

4 Finite Rate Feedback Bit Partitioning in TVC

In this section, the effect of the proposed bit allocation with interference grading for channels which are propagating in time varying environment is derived. If a channel of interest is time varying and experiences a large delay before it reaches base stations, the channel state information becomes out-dated and beamforming to reduce the interference might not really reduce the interference. Since the channel is time varying, to model the time varying behaviour, first order Gauss-Markov model is used. In Gauss–Markov model, the channel temporal correlations are modelled as a function of delay [8]. The channel between the (l, i)th user and the tagged BS is given by

$$h_{l,i,i}[p] = \eta_{l,i,i} h_{l,i,i}[p-1] + \sqrt{1 - \eta_{l,i,i}^2} w_{l,i,i}[p] \tag{19}$$

where $\eta_{l,i,i} \in (0, 1]$ is the fading correlation coefficient, $h_{l,i,j}[p]$ and $w_{i,j}[p]$ is already defined in the Eq. (1). The parameter p denotes that the channel is realized in pth instant of time. The value of $\eta_{l,i,i}$ is modelled by jakes model and is given as $\eta_{l,i,i} = J_0(2\pi f_d^{li} T_F)$ where J_0 is zeroth order Bessel function, f_d^{li} is the maximum Doppler frequency of the user(l, i), and T_F is the frame duration.

Next we derive the necessary equation for the bits required to nullify the interference in time varying channels. Now let us define $\tau_{i,i}$ is the feedback update period for $h_{l,i,i}[p]$. Assume that $\{\tau_{l,i,i} | l = 1, 2, \ldots, L\}$ and $\{b_{l,TVC} | l = 1, 2, \ldots, L\}$, the average sum rate of the user with quantized CDI of time varying channels using Coordinated zero forcing beamforming is expressed as

$$E_{TVC}[R] = E \left\{ \log_2 \left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha} \left\| h_{l,i,i}[\tau_{l,i,i}] \right\|^2 \left| \tilde{h}_{l,i,i}^H w_{l,i}[\tau_{m,i,i}] \right|^2}{\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1} \right) \right\} \tag{20}$$

In the above Eq. (20), the feedback update period $\tau_{l,i,i}$ to realize the channel is also varied to study the rate offset. The upper bound of residual interference (both IUI and ICI) by adapting the similar procedure in deriving the Eqs. (5) and (6) is

$$\tilde{I}_{IUI} + \tilde{I}_{ICI} = \left(E_{l,i,i}(1+r_i)^{-\alpha}(1+\varepsilon)\left\|h_{l,i,i}[\tau_{l,i,i}]\right\|^2 \sin^2\theta_{l,i,i} \sum_{m=1,m\neq l}^{L} \left| q_{m,i,i}^H[\tau_{m,i,i}] w_{m,i}[\tau_{m,i,i}] \right|^2 \right) \tag{21}$$

Substituting the value of $\tilde{I}_{IUI} + \tilde{I}_{ICI}$ from the previous Eq. (21), the sum rate becomes

$$E_{TVC}[R] = E\left\{ \log_2\left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}[\tau_{l,i,i}]\right\|^2 \left|\tilde{h}_{l,i,i}^H w_{l,i}[\tau_{m,i,i}]\right|^2}{\left(E_{l,i,i}(1+r_i)^{-\alpha}(1+\varepsilon)\left\|h_{l,i,i}[\tau_{l,i,i}]\right\|^2 \sin^2\theta_{l,i,i} \sum_{m=1,m\neq l}^{L} \left| q_{m,i,i}^H[\tau_{m,i,i}] w_{m,i}[\tau_{m,i,i}] \right|^2 \right) + 1} \right) \right\} \tag{22}$$

In order to quantify the value of residual interference in time varying channels, the value of $\|h_{l,i,i}[\tau_{l,i,i}]\|^2 \sin^2\theta_{l,i,i} \sum_{m=1,m\neq l}^{L} |q_{m,i,i}^H[\tau_{m,i,i}] w_{m,i}[\tau_{m,i,i}]|^2$ is to be known. Note that $E\|h_{l,i,i}[\tau_{l,i,i}]\|^2 = M$ and $|q_{m,i,i}^H[\tau_{m,i,i}] w_{m,i}[\tau_{m,i,i}]|^2$ is beta distributed with parameter $\beta(1, M-2)$. Further that $E\left(\sin^2\theta_{l,i,i}\right) \prec 2^{-\frac{b_{l,TVC}}{M-1}}$, and from Ref. [17], the quantization error is bounded by

$$\left\|h_{l,i,i}[\tau_{l,i,i}]\right\|^2 \sin^2\theta_{l,i,i} \sum_{m=1,m\neq l}^{L} \left| q_{m,i,i}^H[\tau_{m,i,i}] w_{m,i}[\tau_{m,i,i}] \right|^2 < \eta_{l,i,i}^{2(\tau_{l,i,i}-1)} \left(\left(\frac{M}{M-1}\right) 2^{-\frac{b_{l,TVC}}{M-1}} - 1 \right) + 1 \tag{23}$$

Since the feedback update period $\tau_{l,i,i}$ also affects sum-rate and if there are L users under a given BS, the sum rate of time varying channels for L users is

$$E_{TVC}[R] = E\left\{ \sum_{l=1}^{L} \log_2\left(1 + \frac{E_{l,i,i}(1+r_i)^{-\alpha}\left\|h_{l,i,i}[\tau_{l,i,i}]\right\|^2 \left|\tilde{h}_{l,i,i}^H w_{l,i}[\tau_{m,i,i}]\right|^2}{\left(E_{l,i,i}(1+r_i)^{-\alpha}(1+\varepsilon)\left[\eta_{l,i,i}^{2(\tau_{l,i,i}-1)} \left(\left(\frac{M}{M-1}\right) 2^{-\frac{b_{l,TVC}}{M-1}} - 1 \right) + 1 \right] \right) + 1} \right) \right\} \tag{24}$$

By taking $P_{l,TVC} = E_{l,i,i}(1+r_i)^{-\alpha}(1+\varepsilon)$ and $\delta_{l,TVC} = E_{l,i,i}\left\|h_{l,i,i}[\tau_{l,i,i}]\right\|^2 \left|\tilde{h}_{l,i,i}^H w_{l,i}[\tau_{m,i,i}]\right|^2$, then the above Eq. (24) after simple algebraic manipulations becomes

$$E_{TVC}[R] = E\left\{ \sum_{l=1}^{L} \log_2\left(1 + \frac{(1+r_i)^{-\alpha}\delta_{l,TVC}}{\left(P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)} \left(\frac{M}{M-1}\right) 2^{-\frac{b_{l,TVC}}{M-1}} \right) + \left(P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right) + 1 \right)} \right) \right\} \tag{25}$$

After following the same optimization defined in Eqs. (8), (9) and (10), the solution to the bits allocation of TVC found to be

$$b_{l,TVC} = (M-1)\left[\log_2\left\{ \left(\frac{P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\frac{M}{M-1}\right)}{P_{l,TVC}\left(1-\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)+1} \right) \left(\frac{1}{(M-1)\upsilon} - 1 \right) \right\} \right] \tag{26}$$

The Slack variable υ in the above Eq. (26) after incorporating the water-Filling algorithm is

$$\upsilon = \left(\frac{1}{(M-1)}\right)\left\{\frac{\left(\prod_l^L\left(\frac{P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\frac{M}{M-1}\right)}{P_{l,TVC}\left(1-\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)+1}\right)\right)^{\frac{1}{L}}}{\left(2^{\frac{B_{res,TVC}}{M-1}}\right)^{\frac{1}{L}}+\left(\prod_l^L\left(\frac{P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\frac{M}{M-1}\right)}{P_{l,TVC}\left(1-\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)+1}\right)\right)^{\frac{1}{L}}}\right\} \tag{27}$$

The value of $B_{res,TVC}$ is found using the algorithm define in the Sect. 3. However, in the algorithm additional parameters like feedback update periods $\tau_{l,i,i}$, $\tau_{l,i,j}$ and channel temporal correlation coefficients $\eta_{l,i,i}$, $\eta_{l,i,j}$ are used to find $B_{res,TVC}$. Since the fading correlation coefficient models the delay, the delay determines the adaptive feedback sharing between the desired BS and interfering BS. The cell average throughput of TVC is compared against the IFBC and the results are plotted in Sect. 6.

4.1 Minimization of Rate Offset in Time Varying Channels

To determine the quality of feedback required to support guaranteed cell average sum-rate, the order of feedback scaling is very important. To characterize the rate loss of Time Varying channels, the difference between the rates supported by TVC and Perfect CSI needs to be known. Using Eqs. (3) and (20) and applying the same procedure for deriving the rate of loss of IFBC, the rate loss of TVC is given as $\Delta R_{l,i}^{TVC} = \log_2\left(\tilde{I}_{IUI} + \tilde{I}_{ICI} + 1\right) = \log_2(1 + res)$. The residual interference in time varying channels from Eq. (22) is formed as $\log_2(1 + res) = \log_2(1 + (E_{l,i,i}(1 + r_i)^{-\alpha}(1 + \varepsilon)\|h_{l,i,i}[\tau_{l,i,i}]\|^2 \sin^2\theta_{l,i,i} \sum_{m=1, m \neq l}^{L} |q_{m,i,i}^H[\tau_{m,i,i}]w_{m,i}[\tau_{m,i,i}]|^2))$. Since the feedback update period $\tau_{l,i,i}$ is also taken into consideration, the value of residual interference for time varying channels when delay associated with limited feedback is modelled using Gauss-Markov model [17] is written as

$$res = \tilde{I}_{IUI} + \tilde{I}_{ICI} = \left(E_{l,i,i}(1+r_i)^{-\alpha}(1+\varepsilon)\left\{\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\left(\frac{M}{M-1}\right)2^{-\frac{b_{l,TVC}}{M-1}}-1\right)+1\right\}\right) \tag{28}$$

From Eq. (28), the rate offset of the user (l, i) bounded to

$$\Delta R_{l,i}^{TVC} \leq \log_2\left(1+\left(P_{l,TVC}\left\{\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\left(\frac{M}{M-1}\right)2^{-\frac{b_{l,TVC}}{M-1}}-1\right)+1\right\}\right)\right) \tag{29}$$

As stated earlier, the rate to be maintained within a $\log_2(\Omega)$ bps/Hz, the loss after some algebraic simplifications, is now simplified to

$$\left(\Omega-\left(1+P_{l,TVC}\left(1-\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)\right)\right) \geq \left(P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\frac{M}{M-1}\right)2^{-\frac{b_{l,TVC}}{M-1}}\right) \tag{30}$$

By applying the feedback bit allocation of TVC derived in the Eq. (27) i.e. substituting the value of $b_{l,TVC}$ from (26), the rate off set reduces to

$$\left(\Omega - \left(1 + P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)\right)\right) \geq 2^{-\frac{B_{res,TVC}}{L(M-1)}} \left(P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right) + 1\right) \times \left(\prod_{l}^{L} \left(\frac{P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\frac{M}{M-1}\right)}{P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right) + 1}\right)\right)^{\frac{1}{L}} \tag{31}$$

The above equation is rearranged to get total feedback bits $B_{res,TVC}$ as

$$B_{res,TVC} = L(M-1)\left\{ \mathrm{Log}_2\left[\frac{\left(P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right) + 1\right)}{\left(\Omega - \left(1 + P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right)\right)\right)}\right] + \mathrm{Log}_2\left(\prod_{l}^{L} \left(\frac{P_{l,TVC}\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\left(\frac{M}{M-1}\right)}{P_{l,TVC}\left(1 - \eta_{l,i,i}^{2(\tau_{l,i,i}-1)}\right) + 1}\right)\right)^{\frac{1}{L}} \right\} \tag{32}$$

The $B_{res,TVC}$ is a function of fading correlation coefficient. From the above equation, the grading factor ε in $P_{l,TVC}$ is multiplied by $\eta_{l,i,i}^{2(\tau_{l,i,i}-1)}$, the number of bits to be scaled to maintain rate off-set is determined by the fading correlation coefficient. In the following numerical results section, the required number of bits in proposed TVC and proposed IFBC are compared for ease of analysis.

5 Numerical Results

In this section, the performance of the proposed schemes are demonstrated to get more insights of the findings derived in the previous sections. The impact of bit allocations between the serving BS and interfering BS on the sumrate is assessed from the simulated results. In addition to the throughput performance, the scaling of bits to achieve the multiplexing gain in both the proposed models are also presented.

A simple one dimensional two cell model with each cell having a radius of about 500 meters is considered for numerical simulation. Throughout the simulation $M = 4$, $\alpha = 3.8$, $L = 2$ and $K = 2$. Unless otherwise specified, the following parameters are used in the model. The Correlation coefficient $\eta_{l,i,i}$ is calculated when the carrier frequency is 2 GHz, the Frame duration T_F is 5 ms and the relative speed between user (l, i) and the BS is assumed to be varying between 0 and 10 km/h. The total feedback bits per user i.e. B_{res}, $B_{res,TVC}$ is fixed at 10 and grading threshold ε_T is fixed at 0.3. The maximum received signal to Noise ratio $\gamma_{l,i,i} = \frac{E_{l,i,i}}{(1+r_i)^{\alpha}}$ on the basis of unit noise power at the user (l, i) is set be 20 dB at a distance $r_i = 500$ m. Since the system is for throughput maximization, the received signal power $\gamma_{l,i,i}$ measured against unit noise power is considered as SNR in this work.

The Fig. 2 shows the feedback partitioning between the serving base stations and interfering base station when the user distance r_i from the serving base station increases. Since the allocation of the bits are primarily to increase the sum-rate, the proposed allocation reduces the region of cooperation between the BSs thereby reducing the overhead of limited feedback. The previous allocation in the Ref. [14] starts allocating more bits in the non-cooperative region to the interfering base stations even the residual inter-cell (ICI) interference is very less compared to residual inter-user interference. That is, it corresponds

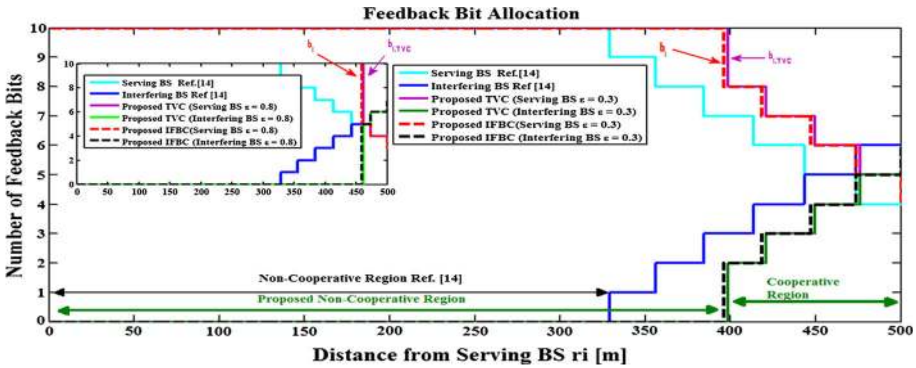

Fig. 2 Feedback allocation strategy for IFBC for various ε when $\gamma_{l,i,i}(r_i = 500) = 10$ dB and $\gamma_{l,i,j}(r_j = 500) = 10$ dB, $\tau_{l,i,i} = \tau_{l,i,j} = 10$

to the ratio $\varepsilon = 0.1$ of the proposed model. The proposed methodology starts the allocation when the ratio ε is greater than the threshold. It can easily be verified from the Fig. 2 that even for $\varepsilon = 0.3$, the serving base stations gets more bits compared to the interfering base stations. Moreover the non-cooperative region of the proposed allocation compared to the other existing schemes is extended. This is very much required whenever the user is in the interior region of the cell where the value of residual ICI is bare minimum compared to the residual IUI.

Moreover, if the grading factor ε is nearing to its maximum value i.e. 1(in the cooperative region it usually happens), both ICI and IUI are equal. In this case the grading ratio ε is greater than ε_{sig} and the total bits are allocated between IUI and ICI as per the steps of algorithm. This is clearly demonstrated in the Fig. 2 by taking $\varepsilon_{\mathrm{sig}}$ as 0.8 and is plotted inside the Fig. 2. Based on the channel conditions and the grading threshold, the bits are allocated in TVC and the actual allocation to the interfering base stations starts at a little later distance. The illustration is given for $\varepsilon_{\mathrm{T}} = 0.3$ and $\varepsilon_{\mathrm{sig}} = 0.8$ in Fig. 2

In Fig. 3, it is very important to note that when ε is in lower range 0.00–0.3 (Non-Cooperative Region), the equal bit partitioning performance is comparable to the proposed scheme. Meanwhile, at the high end of ε, the rate of equal bit partitioning drops to half of the rate. This is due to the half of the feedback bits are allocated to residual ICI. The proposed scheme maintains the higher rate even in cooperative region because of the grading factor ε which allocates larger feedback bits to the interference which is higher in value compared to the other interference. The serving BS in turn coordinates with the neighbouring base stations to keep the ICI to a minimum value. As expected because of channel temporal correlations, the performance of the proposed TVC scheme is slightly lower than the IFBC Scheme.

It can be easily observed from the Fig. 4 that the minimum rate loss occurs when when the sub-frame index time matches with the Least common multiple of $\tau_{l,i,i}$ and $\tau_{l,i,j}$. Since IFBC is independent of feedback duration, the rate loss is independent of sub-frame index. The proposed IFBC and TVC loss are around 20% lesser in non-cooperative region and it reduces to around 5% in the cooperative region compared to previously reported rate loss in Refs. [14] and [17].

The impact of proposed feedback bit allocation of TVC and IFBC on achievable rate for fixed total residual feedback bits B_{res}, $B_{res,TVC}$ are shown in Fig. 5. The proposed IFBC scheme with $\varepsilon_T = 0.3$ achieves approximately 20% higher throughput compared to the

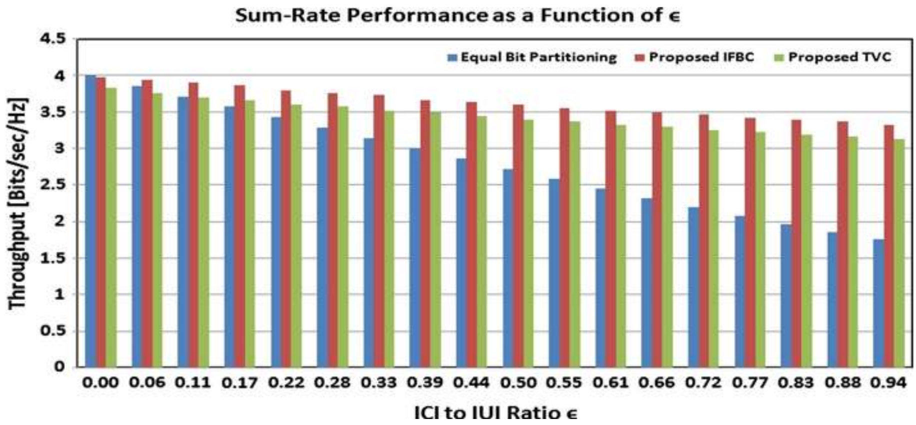

Fig. 3 Sum-rate against ε for $B_{res} = B_{res,TVC} = 8$, $\tau_{l,i,i} = \tau_{l,i,j} = 2$, $\gamma_{l,i,i}(r_i = 500) = 10$ dB and $\gamma_{l,i,j}(r_j = 500) = 10$ dB

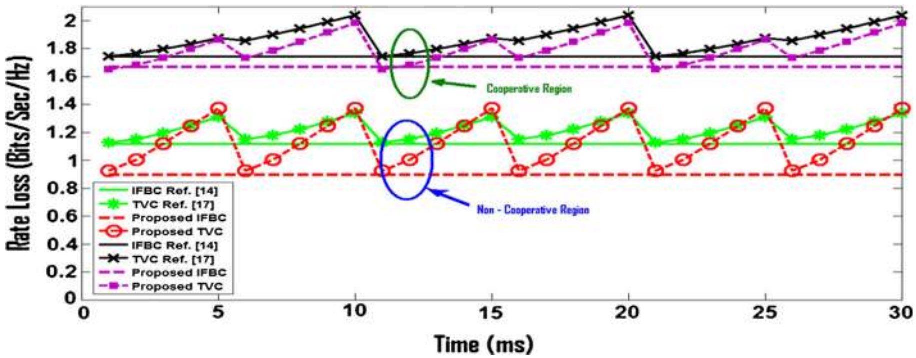

Fig. 4 Rate loss versus sub-frame index T_s when $B_{res} = B_{res,TVC} = 8$, $\tau_{l,i,i} = 5$, $\tau_{l,i,j} = 10$, $\eta_{l,i,i} = -\eta_{l,i,j} = 0.99$ and $\gamma_{l,i,i}, \gamma_{l,i,j}(r_i, r_j = 500) = 10$ dB

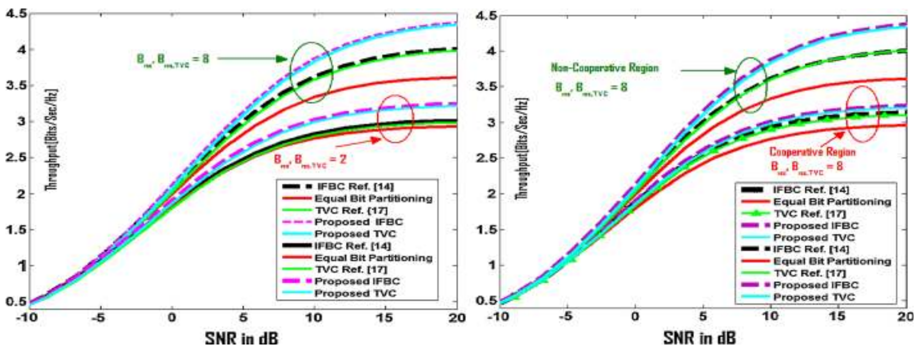

Fig. 5 Sum-rate performance for fixed B_{res}, $B_{res,TVC}$ when $\tau_{l,i,i} = \tau_{l,i,j} = 2, \eta_{l,i,i} = \eta_{l,i,j} = 0.99$

equal bit allocation scheme and it achieves 15% gain in throughput over the previous schemes of Refs. [14] and [17] if B_{res}, $B_{res,TVC} = 2$. When we increase ε to 0.8 i.e. it becomes ε_{sig}, because of stronger ICI, the proposed scheme performs slightly better than that of equal bit and previous allocation scheme i.e. around 1–5% throughput gain. This is due to the fact that the IUI is more dominating in the non-cooperative region. The number of bits required to nullify the effects of interference is less if total bits are fixed at B_{res}, $B_{res,TVC} = 2$. The interesting observation from Fig. 5 is that as the residual feedback bit B_{res}, $B_{res,TVC}$ increases, the proposed allocation produces a marked improvement over equal bit allocation and the previous schemes of Refs. [14] and [17]. The throughput improvement of about 35% over equal bit allocation in both IFBC and TVC is observed in the edge of non-cooperative region. Moreover the plot also compares the performance of proposed scheme in both ends of cooperative and non-cooperative region. When B_{res}, $B_{res,TVC}$ is fixed (B_{res}, $B_{res,TVC} = 8$), the cell average sum throughput is decreased in the cooperative region compared to the non-cooperative region. This reduction is due to the increased ICI at the cooperative region and the feedback bits are shared between IUI and ICI.

The sum rate performance of the proposed TVC, IFBC at a lower SNR (SNR = −0.5 dB) in both the cooperative and non-cooperative regions with increasing feedback duration is almost equal to the previous schemes reported in Refs. [17] and [14] respectively. In middle and high SNR regimes (SNR = 5, 10 dB), the proposed schemes achieves around 10–15% throughput improvement over the previous schemes if the feedback update duration is higher. This necessitates that the feedback needs to be updated frequently to yield larger sum rate and the results are shown in Fig. 6. As predicted the IFBC is not affected by feedback duration.

To verify the scaling of Feedback bits in TVC and IFBC, the cell average sum rate performance with increasing number of feedback bits B_{res}, $B_{res,TVC}$ is evaluated. It can easily be inferred from the plot of Fig. 7, the proposed scheme maintains a rate off-set of $\mathrm{LLog}_2\,(\Omega)$ for different ε. The proposed IFBC and TVC are tested with $\varepsilon = 0.3$. If $\Omega-1$ equals 2 and L = 2, the TVC and IFBC maintains a rate off-set of 3.12 bps/hz. The rate offset of $\mathrm{Log}_2\,(\Omega)$ corresponds to a power offset of 3 dB i.e. it is equivalent to M $\mathrm{Log}_2\,(\Omega)$ bps/Hz [1]. Theoretically the proposed scheme maintains a power off-set of 6 dB [L (M−1) $\mathrm{Log}_2\,(\Omega-1) = 6$ dB when L = 2, M = 4 and $\Omega-1 = 2$]. From the plot, the simulated results of both TVC and IFBC maintain a power off-set less than 5 dB. But at high SNR, TVC requires more scaling to maintain the same rate loss. However, to maintain a lower power offset i.e. for example to maintain 1 dB, the proposed TVC and IFBC require additional 1.95 L (M−1) bits only.

From Ref. [1], the throughput curve achieves a multiplexing gain of $M\left(\frac{s}{M-1}\right)$ if B_{res}, $\mathrm{B_{res,TVC}}$ are scaled as $B_{res} = s\ \log{}_2 P$ where s is some constant. The proposed allocation of IFBC scales as 0.6 L (M−1) i.e. it achieves a multiplexing gain of $M\left(\frac{0.6(M-1)L}{M-1}\right) = M(0.6)L$. The achieved multiplexing gain of 4.8 bps for L = 2 and M = 4 corresponds to a power off-set of 7.5 dB. The scaling in TVC requires additional 0.5 dB i.e. 8 dB to achieve the same multiplexing gain as that of IFBC. These results are demonstrated in Fig. 8.

But at the same time both IFBC and TVC achieves full multiplexing gain at higher scaling. That is, if it is scaled 1.5(M−1) L times, then full multiplexing gain is achieved. Higher scaling closed the SNR gap between Zero-forcing Ref. [1] and proposed IFBC at lower value of SNR. The gap relative to zero-forcing and proposed TVC approaches to

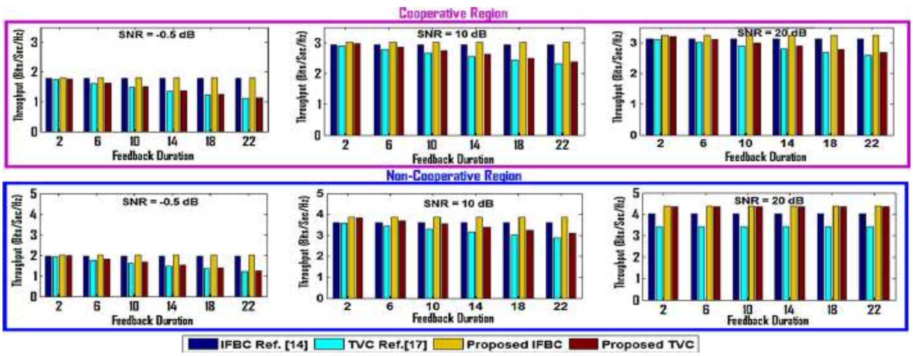

Fig. 6 Sum-rate performance versus feedback duration $\tau_{l,i,i}$ $(\tau_{l,i,i} = \tau_{l,i,j})$ when $B_{res} = B_{res,TVC} = 8$, $\eta_{l,i,i} = \eta_{l,i,j} = 0.99$

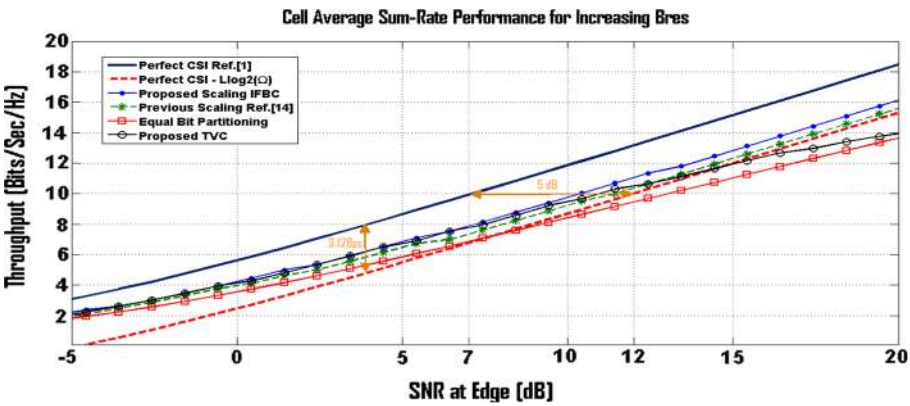

Fig. 7 Sum-rate performance with increasing number of total feedback bits $(B_{res}, B_{res,TVC})$ when $\tau_{l,i,i} = \tau_{l,i,j} = 2$, $\eta_{l,i,i} = \eta_{l,i,j} = 0.99$

Zero only at very high SNR. This is primarily due to channel correlation effects and these results can easily be verified from the plot of Fig. 8.

The Fig. 9 Illustrates the throughput performance with increasing number of bits if feedback update duration and target SNR varies simultaneously. In IFBC the channel conditions are constant during a frame period and if the feedback update duration varies, the achievable throughput is not affected. This proved that the scaling of feedback bits is independent of update period provided the channel is constant throughout the frame duration. But at the same time, for TVC, to achieve the guaranteed throughput, the feedback should necessarily be scaled with respect to both the target SNR and update period. Practically the channel status needs to be frequently updated in TVC to realize the required Quality of Service.

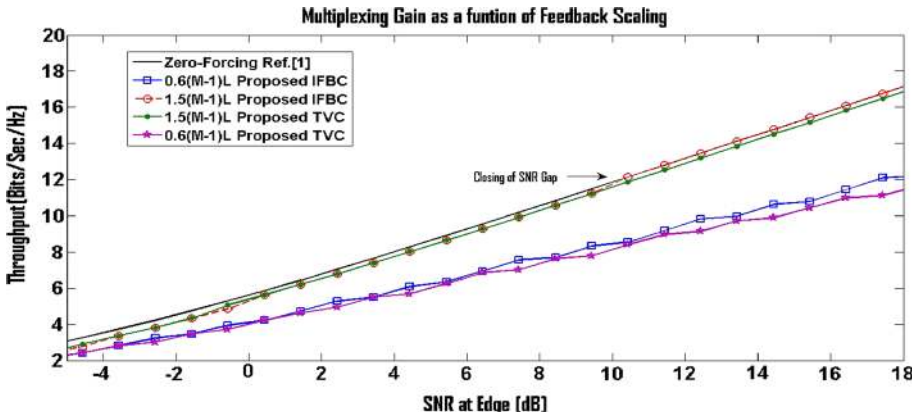

Fig. 8 Feedback scaling for multiplexing gain when $\tau_{l,i,i} = \tau_{l,i,j} = 2$, $\eta_{l,i,i} = \eta_{l,i,j} = 0.99$

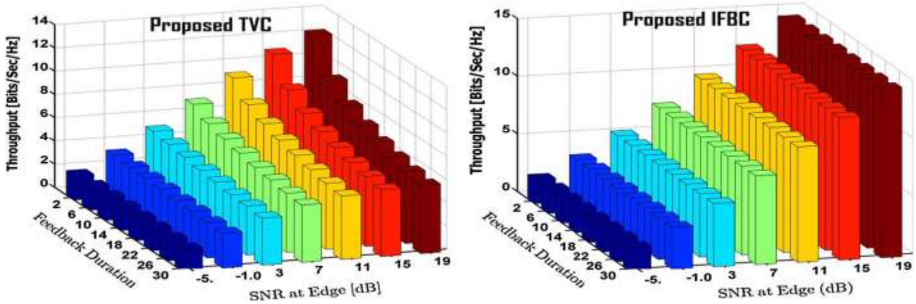

Fig. 9 Sum rate performance with simultaneous variations of $\tau_{l,i,i}$ ($\tau_{l,i,i} = \tau_{l,i,j}$) and $\gamma_{l,i,i}$ ($\gamma_{l,i,i} = \gamma_{l,i,j}$) when $\eta_{l,i,i} = \eta_{l,i,j} = 0.99$

6 Conclusion

In this paper, a sum rate loss performance is characterized in IFBC and TVC and proposed a feedback bits sharing scheme between interfering and desired channel in order to nullify the effect of significant interference in average throughput. The proposed scheme minimizes the quantization error by optimally fitting the given feedback bits per user and keeping the individual user feedback budget constant. Numerical results proved that the healthy improvement of sum rate performance under this proposed feedback scheme when fixing threshold for interference grading. Finally we have shown that to keep a constant rate loss with a determined power offset and to achieve multiplexing gain, the total feedback bit is scaled linearly with feedback update period and SNR which in-turn is a function of the path loss, the number of users, the number of antennas in addition to fading correlation coefficient in TVC. Further extension to this work includes, incorporating channel estimations errors in allocating the feedback budget.

References

1. Jindal, N. (2006). MIMO braodcast channels with finite rate feedback. *IEEE Transactions on Information Theory, 52*(11), 5045–5060.
2. Yoo, T., Jindal, N., & Goldsmith, A. (2007). Multi-antenna downlink channels with limited feedback and user selection. *IEEE Journal of Selected Areas in Communications, 25*(7), 1478–1491.
3. Au-Yeung, C. K., & Love, D. J. (2007). On the performance of random vector quantization limited feedback beamforming in a MISO system. *IEEE Transcations on Wireless Communicationn, 6*(2), 458–462.
4. Cho, S., Huang, K., Kim, D., & Seo, H. (2012). Interference alignment for uplink cellular systems with limited feedback. *IEEE Communication Letters, 16*(7), 960–963.
5. Zhang, J., & Andrews, J. G. (2010). Adaptive intercell interference cancellation in multicell wireless networks. *IEEE Journal on Selected Areas in Communications, 28*(9), 1455–1468.
6. Gesbert, D., et al. (2010). Multi-cell MIMO cooperative networks: A new look at interference. *IEEE Journal on Selected Areas in Communication, 28*(9), 1380–1408.
7. Balaji, S., Santhakumar, R., Mallick, P. S., & Nithiyanandan, L. (2013). Intercell interference nulling with bit partitioning for extended Clarke's modelled multi-cell wireless networks. *Australian Journal of Electrical and Electronics Engineering, 10*(4), 535–541.
8. Bhagavatula, R., & Heath, R. W. (2011). Adaptive bit partitioning for multicell intercell interference nulling with delayed limited feedback. *IEEE Transactions on Signal Processing, 59*(8), 3824–3836.
9. Lin, Y.-P., et al. (2012). Bit allocation and statistical precoding for correlated MIMO channels with limited feedback. *IEEE Transactions on Vehicular Technology, 61*(2), 597–606.
10. Kim, K., Kim, T., Love, D. J., & Kim, H. (2012). Differential feedback in codebook-Based multiuser MIMO systems in slowlying varying channels. *IEEE Transactions on Communications, 60*(2), 578–588.
11. Sohn, I., Park, C. S., & Lee, K. B. (2012). Downlink multiuser MIMO systems with adaptive feedback rate. *IEEE Transcations on Vehicular Technology, 61*(3), 1445–1451.
12. Kim, S., Kim, D., & Lee, Y. H. (2013). Limited channel feedback for coordinated beamforming under SINR requirements. In *IEEE International Conference on Communications (IEEE- ICC'13)* (pp. 431–435).
13. Akom, S., & Heath, R. W., Jr. (2013). Interference coordiantion: Random clustering and adaptive limited feedback. *IEEE Transcations on Signal Processing, 61*(7), 1822–1834.
14. Lee, Namoon., & Shin, Wojnae. (2011). Adaptive feedback scheme on K-cell MISO interfering broadcast channel with limited feedback. *IEEE Transcations on Wireless Communications, 10*(2), 401–406.
15. Ozbek, B., & Le Ruyet, D. (2014). Adaptive limited feedback links for cooperative multi-antenna multicell networks. *EURASIP Journal on Wireless Communication and Networking, 1,* 1–11. doi:10.1186/1687-1499-2014-193.
16. Balaji, S., Santhakumar, R., & Mallick, P. S. (2015). Enhancing coverage and rate of cell edge user in multi-antenna poission voronoi cells. *Journal of Circuits, Systems and Computers*. doi:10.1142/S0218126615501170.
17. Kim, H., Yu, H., & Lee, Y. H. (2015). Limited feedback for multi-cell zero-forcing coordinated beamforming in time-varying channels. *IEEE Transcations on Vehicular Technology, 64*(6), 2349–2360.
18. Balaji, S., Mallick, P. S., & Gnanagurunathan, G. (2015). Threshold based limited feedback bit partitioning for interference channels in multicell systems. *Journal of Engineering Science and Technology Review, 9*(4), 82–89.
19. Hassen, W. B., Afif, M. & Tabbane, S. (2016). Interference mitigation through inter-cell interference coordination using virtual PRB allocation in 4G networks. *Wireless Personal Communication*, *90*(3), 1179–1209.
20. Muharar, R., Zakhour, R. & Evans, J. (2014). Base station cooperation with feedback optimization: A large system analysis. *IEEE Transactions on Information Theory*, *60*(6), 3620–3644.
21. Annapureddy, V. S. & Veeravalli, V. V. (2009). Gaussian interference networks: Sum capacity in the low-interference regime and new outer bounds on the capacity region. *IEEE Transactions on Information Theory*, *55*(7), 3032–3050.
22. S. Balaji, & Mallick, P. S. (2017). Performance analysis of adaptive limited feedback coordinated zero-forcing multicell systems in time-varying channels: A practical approach. *International Journal of Communication Systems*, *30*(14):e3301.
23. Boyd, S., & Vandenberghe, L. (2004). *Convex optimization*. UK: Cambridge University Press.

Dr. S. Balaji received Master of Technology from National Institute of Technology, Tiruchirappalli, and Ph.D. in Electrical Engineering from VIT University, Vellore, India. Currently he is working as Associate Professor in the School of Electrical Engineering (SELECT), VIT University, India. He has published more than 21 research papers in the area of wireless communication in International Journals, National Level Journals and Conferences. His areas of interest include MIMO, OFDM, and Wireless Communication with emphasis on Signal Processing.

Dr. P. S. Mallick (M'03, SM'10) received M.S. degree from the University of Chittagong, Bangladesh and Ph.D. from Jadavpur University, India. He worked 4.5 years in a Sweden based electronics industry named IAAB Electronics as a Technical Head. He has 10 years of Teaching experience where he led various research teams and developed "Online Lab in Microelectronics", "Monte Carlo Simulator", and "Electric Fencer". His current area of research interest includes Optoelectronic materials and Devices, Nanoscale CMOS and Nanoelectronics. He has published more than 75 research papers in different Journals and Conferences of International repute and authored a book on Matlab and Simulink. At Present Dr. P. S. Mallick is working for School of Electrical Engineering, VIT University, Tamil Nadu, India, as a Senior Professor. He has received the prestigious Jawaharlal Nehru Scholarship in 1998 for his doctoral research work. He is one of the enlisted technical innovators of India in 2007.